

Hamiltonian or not with all Possible Paths

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Abstract

Given a Graph $G(V, E)$, We Consider the problem of deciding whether G is Hamiltonian, that is- whether or Not there is a simple cycle in E spanning all vertices in V . [1] However to Verify that the given cycle is Hamiltonian by checking whether it is permutation of the vertices of V and whether each of the consecutives edges along the cycle actually exists in the Graph. This Verification Algorithm can certainly be implemented to run in $O(n^2)$ time, where n is the length of the encoding of G [2]. But to predict in Advance that the Graph has Hamiltonian Cycle or not was still Exponential before this Algorithm. This Problem is known to be NPComplete hence cannot be solved in Polynomial time in $|V|$ unless $P=NP$. However till today there was no known Criterion we can apply to determine the existence Hamiltonian Circuit in General [3]. For its Exponential time We can Refer to theorems: - Vertex Cover problem is polynomially transformable to the Hamiltonian circuit Problem for Directed graphs, hence the Hamiltonian Circuit problem for Directed Graph is NP-Complete and the Hamiltonian Circuit Problem for Directed Graph is Polynomially transformable to Hamiltonian Cycle Problem for Undirected Graph, hence the Hamiltonian Cycle Problem for undirected Graph is NP-complete [4]. Note that these derivations are based on the CNF-Satisfiability. Through this Paper we have introduced a Newer Algorithm with different approach to determine whether a given Graph is Hamiltonian or Not with all possible Paths, by applying Few Mathematical and logical Operations. This provides necessary and sufficient condition for a graph to be Hamiltonian.

Index terms— Adjacency matrix, Adjacency List, Nodes, Vertices, Edges, Hamiltonian circuit.

1 Efficient Algorithm to Determine Whether a given

Graph is Hamiltonian or not with all Possible Paths Narendra Pratap Singh¹, Ramu Agrawal² & Indra Paliwal³ Abstract -Given a Graph $G(V, E)$, We Consider the problem of deciding whether G is Hamiltonian, that is-whether or Not there is a simple cycle in E spanning all vertices in V . [1] However to Verify that the given cycle is Hamiltonian by checking whether it is permutation of the vertices of V and whether each of the consecutives edges along the cycle actually exists in the Graph. This Verification Algorithm can certainly be implemented to run in $O(n^2)$ time, where n is the length of the encoding of G [2]. But to predict in Advance that the Graph has Hamiltonian Cycle or not was still Exponential before this Algorithm. This Problem is known to be NP-Complete hence cannot be solved in Polynomial time in $|V|$ unless $P=NP$. However till today there was no known Criterion we can apply to determine the existence Hamiltonian Circuit in General [3]

2 Introduction

Hamiltonian Problem is Decision Problem in which $G(V, E)$ should be traversed from any one vertex to same vertex without repeating any vertex again (means, Vertex should be traverse exactly once).

We look for a long sequence of vertices $v_0, v_1, v_2, \dots, v_{n-1}$ visit all vertices in V such that $i \neq 0, (v_i, v_{(i+1) \bmod n}) \in E$, along with the element of Adjacency Matrix $A_i, j = 1, \text{if } (j, i) \in E$, 0, otherwise.

From the general prediction as prescribed in literature that Hamiltonian Cycle exists if and only if there is an n -long Author : Department of Computer Science, BSA College of Engineering and Technology, Mathura, U.P, India. E-mail : narendrapratapbsa@gmail.com E-mail : ramuagrawalbsa@gmail.com E-mail : indrapaliwal@gmail.com tour that cover all the vertices and returns to the starting point.

Scientist around the globe deduced the Method based on the number of edges and degrees of graph, some for planarity and some for connected but they all failed for a general graph and was not sufficient. It follows the CNF-satisfiability also [4].

But we put through the above statement from the mathematical and logical point of view.

By this algorithm, now the scientist will have reasonable condition to determine the Hamiltonian Circuit in Advance without traversing it vertex to vertex manually on the paper.

Till today this problem which spurred the computer scientist around the globe to be able to draw an Algorithm which Culminate the possibilities, the usage of global information was shown to speed up the process: however it has cost in communication and complexity of individual agent. Now in our Algorithm there is no foundation for an undirected, directed, planarity, colorability, and connectedness of a graph, it can be applied to the all types of graphs.

Rest of the paper is organized as follows. Section2 present the related work. The proposed method algorithm has been described in section3. In section4, experimental results and sample run have been presented and paper is concluded in section5.

3 II.

4 Related work

Since, its (Hamiltonian Cycle Problem) origin, by famous Irish Mathematician Sir William Rowan Hamilton, 1859, was still unsolved. There was no known criterion we could apply to determine the existence of Hamiltonian circuit in general. A circuit in a connected graph G is said to be Hamiltonian if it includes every vertex of G . Hence a Hamiltonian Circuit in a Graph of n vertices cost of exactly n edges. Obviously, not every connected Graph has Hamiltonian Circuit. For example, neither of the Graph shown in figures (2.1 and 2.2) and has a Hamiltonian circuit. This raise the Question: What is the necessary and sufficient condition for a connected Graph G to have Hamiltonian Circuit? [5] Also, No known Characterization to determine Hamiltonian graph in any given Graph G has been found [6]. (D D D D) C 2012

5 Year

However several Scientist has proposed several methods on the basis of degree and edges with the reference to any specific graph (like connected, planarity, etc.). But they had not found full success with necessary condition to predict Hamiltonian cycle in advance for every Graph. Most of works has been presented before the 1975. Hence there was no programming based algorithmic approach had been considered? Some famous works are as follows:

? Every Graph G with $3 \leq n$ vertices and minimum degree at least $n/2$ has a Hamiltonian Cycle [7] (Dirac 1956).

[Note that this theorem bound prediction within limit of The following theorem characterizes all Hamiltonian Sequences.3 ? n .] ? Every Graph G with $|G| \geq 3$ and $K(G) \geq 2$

? (Chvatal 1972), An integer Sequence $(a_1, \dots, a_n)$ such that $0 \leq a_1 \leq a_2 \leq \dots \leq a_n < n$ and $n \geq 3$ is

Hamiltonian if and only if the following holds for every $i < n/2$: [11] ? An integer sequence $(a_1, \dots, a_n)$ such that $n \geq 2$ and $0 \leq a_1 \leq a_2 \leq \dots \leq a_n < n$ is path Hamiltonian if and $a_i \leq i \leq a_{n-1} \leq n-i$ only if every $i \leq n/2$ is such that $a_i < i \leq a_{n+i-2} \leq n-1$

Hamiltonian Cycle in the square of a graph For $k=1$, this is precisely Dirac's theorem the case $k=2$ had already been conjecture by Posa in 1963 and was proved for large n by Kamlos, Sorojy & Szemerdi, 1996. [13] Beyond the above given thesis, the age comes to programming and computer scientist developed several algorithms in the same context but did not get succeed to make sure exact prediction for a graph to be Hamiltonian or not. These are given as follows: (Fleischner 1974), if G is a 2-Connected

? 1. Vertex ant walk (VAW) has the Hamiltonian cycles as its limit cycle; however we do know if those are the only limit cycles of the process which are longer than the n variables.

6 A probabilistic Version of VAW rule does not

determine the next neighbor specifically, but assigns each neighbor a probability according to its current (μ, t) mark (e.g) the probability of jumping from u to v be $\text{Prob}(u \rightarrow v) = (1/(1+\mu(v)))/(\sum_w N(u) (1/(1+\mu(w)))$

Where, clearly, $\sum_w N(u) \text{prob}(u \rightarrow w) = 1$ $N(u)$ stands for the sets of vertices $v \in V$ such that $(u, v) \in E$

). In such a semi-Random process faster, (In recognizing a Hamiltonian Graph), on the average, or lower than the deterministic one? [Note that, the prediction in above method is probabilistic, means of having some amount of Uncertainty.] ? Solving the Hamiltonian cycle Problem using symbolic determinant by V.Ejov ?

99 Bitonic Euclidean traveling-salesman problem [2] The Euclidean traveling-salesman problem is the problem of
 100 determining the shortest closed ©

101 7 The proposed method

102 In this section, we will present our original approach to determine all Hamiltonian paths for a Given Graph G.
 103 $G(V, E)$ is the ordered pair consists two sets, V for vertices($v_1, v_2, v_3, \dots, v_n$) and E for edges (e_1, e_2 , Algorithm
 104 (for First Basic method*):

105 1. Draw the Adjacency matrix (Inputting graph):

106 Let take adjacency matrix for any undirected graph without parallel edges or loops. Adjacency matrix for
 directed/undirected having loops and parallel edges. ^{1 2}



Figure 1:

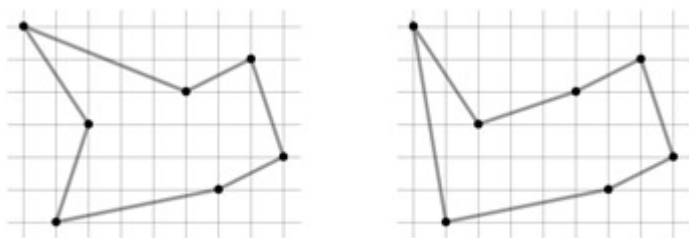


Figure 2: Year

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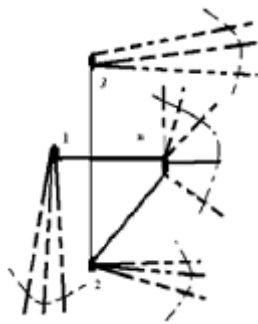
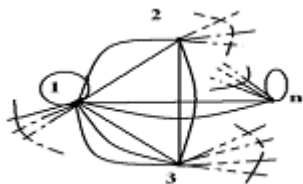


Figure 3:



31

Figure 4: Fig. 3 . 1 :

3233

	1	2	3	4
1	0	1	0	0
2	1	0	1	1
3	0	1	0	0
4	0	1	0	0
5	0	1	1	1
6	1	0	0	0

Figure 5: Fig. 3 . 2 :Fig. 3 . 3 :

[Note: . For its Exponential time We can Refer to theorems: -Vertex Cover problem is polynomially transformable to the Hamiltonian circuit Problem for Directed graphs, hence the Hamiltonian Circuit problem for Directed Graph is NP-Complete and the Hamiltonian Circuit Problem for Directed Graph is Polynomially transformable to Hamiltonian Cycle Problem for Undirected Graph, hence the Hamiltonian Cycle Problem for undirected Graph is NP-complete[4]. Note that these derivations are based on the CNF-Satisfiability. Through this Paper we have introduced a]

Figure 6:

.1 Year

Do the same operation for all the sequences, we get $P_1, P_2, P_3, \dots, P_n$.

.2 B. Then we apply logical OR(||) operator between all

the path values. *fortunately this algorithm provides sufficient conditions for a Graph to be Hamiltonian and also print all the possible paths but Unfortunately takes more time for large Graphs, so we were in search for next Algorithms which take less time than this, therefore we developed second Algorithm which facilitate all the requirements.

We prepare second Algorithm using the concept of Graph theory, means of having linked list, structures and few pointers.

If all nodes are reachable from A, than flights may seek to any Airport, Note that this step will make just one comparison and proceed to next one (say, B).

On the next point, it will decide to move on the next Airport Except the later traversed Airport (A). If this node is reachable to another then it will move on by making just single Comparison again. This sequence of comparison at one time and moving on the next node will remain next Node will remain continue till it did not reach at the initial vertex (Airport).

.3 $T(n) = 1+1+1+??+1$

(n times) $T(n) = n T(n)$ For Worst Case:

In the worst case, the Airplane will choose the move to specific Airport by making (n-1) comparison for itself; therefore the airplane at the next step will decide the next move to further proceeding after making n-1 comparison. The routine will remain till the Airplane did not reach at the initial Airport (or Standing point).

$= n(n-1)!$ Hence, this Algorithm runs near to polynomial for small vertices sets.

V.

.4 Conclusion

Our method exploits the ways to find Hamiltonian Circuit using Sequential and predicate logic and opens up opportunities for future researcher interested in this problem succinctly in much advance way. However our method has successfully find out all possible paths but still working near to polynomial not exactly polynomial, but it opens opportunities to think about this problem by applying advance methods of predicate calculus to find out all possible paths in polynomial time.

[Seymour ; T et al. ()] , P D P Seymour ; T , V C Mc Donough , Marron . 1974. OD) Combinatorics, Cambridge University Press. 3.

[Thomassen ()] , C Thomassen . *J.Graph* 1983. 7 p. .

[Fleischner and Theory ()] , J Combin Fleischner , Theory . 1991. p. .

[Horowitz et al. ()] , Ellis Horowitz , Sataj Sahni , Sanguthevar Rajshekharan . 2009.

[Israel et al.] *An Ant Inspired Heuristic for Recognizing Hamiltonian Graphs*, A Israel , Alfred M Wagner , Bruckstein . p. .

[Bondy U and Murthy ()] J A S R Bondy & U , Murthy . *Graph Theory With Applications*, (Macvmillan) 1976.

[Cormen et al. ()] T H Cormen , C E Leiserson , R L Rivest . *Introduction to Algorithms*, (Cambridge, MA) 2009. MIT Press.

[Diestel ()] Reinhard Diestel . *Graph Theory Electronic edition*, (New york) 2000. 2000. Springer-verlog. p. .

[Bollobas ()] *External Graph Theory*, B Bollobas . 1978. Academic Press.

[Deo (20007)] *Graph Theory with Application to Engineering and Computer Science*, Narsingh Deo . 20007. Prentice Hall India. p. .

[Halin ()] R Halin . *Graph theorie*, Wissen Schatliche Buchgessellschaft, 1980.

[Komlos et al. ()] J Komlos , G N Sarkozy , & E Szmredi . *Random Structures and Algorithms*, 1976. p. .

[Aho et al. ()] *The Design and Analysis of Computer Algorithms*, Alfred V Aho , John E Hopcroft , Jefferey D Ullman . 2008. p. .