A New Method for Gray Level Image Thresholding Using Spatial Correlation Features and Ultrafuzzy Measure

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Abstract

One of the most recent techniques employed to estimate an optimal threshold of a gray level image for segmentation is ultrafuzzy measures. In this paper, we introduce relative fuzzy membership degree (RFMD) taking spatial correlation among the pixels in the image into account. We also propose a novel thresholding technique by combining two-dimensional histogram, which was determined by using the gray value of the pixels and the local average gray value of the pixels using ultrafuzziness and RFMD. Compared to fuzzy membership degree, RFMD of type-II fuzzy sets and ultrafuzzy measure is able to better segment critical gray level images. It was observed that the outcome is so encouraging in objective and subjective perspectives over the existing method for all varieties of images.

Index terms— type-i fuzzy, type-ii fuzzy, ultra fuzziness, relative gray value.

Introduction ltimate aim of image processing is object recognition and extraction from the scene. In this regard, image segmentation become paramount in many computer vision and image processing applications for further analysis of the foreground objects in order to explore the features. Thresholding approach is the simplest and well-known technique for image segmentation.

Accuracy of segmentation depends upon the process which is adopted. It is essential to fin optimal threshold value to group the image pixels into two well defined and non-overlapping subsets, representing image foreground and background. In general, histogram of an ideal image has a deep valley between two peaks. In pursuit of the threshold, valley region is the best place to search in bimodal histogram images because both the peaks, in most of the cases, represent the object and background but this criterion may not be suitable for all types of images.

Image segmentation plays a vital role in analysis of objects extracted from background in many image processing applications. The application areas such as document image processing, scene or map processing, satellite imaging and automatic material inspection in quality control tasks are some of the example that employ image thresholding to extract useful information from images. Medical image processing is another specific area that has tremendously using image thresholding to help the experts to better understand digital images for a more accurate diagnosis and to plan appropriate treatment.

Image segmentation based on gray level histogram thresholding is regarded as a two-class clustering approach to divide an image into two regions; object and background. Basically image thresholding can be considered as two types; one is global thresholding and other is local thresholding. If a single threshold value is applied for entire image to segment, pixels whose gray level is under this value are assigned to one region and the remainder to the other. Images are, generally, classified into unimodal, bimodal and multimodal depending on their histogram shapes. When the histogram doesn't exhibits a clear separation between two peaks ordinary thresholding techniques might under perform. Hence there is a demand for a robust methodology to binarise all types of images as specified above. Fuzzy set theory provides better convergence when compared with non-fuzzy methods. This paper records an automated approach using spatial correlation introduced in a novel way with fuzzy S-function and image ultrafuzziness as a fuzzy measure without using an entropic criterion function.

In case of ideal images the image histogram shows a deep valley between two distinct peaks, each one represents either an object or background and the threshold falls in the valley region. But in case of unimodal and bimodal images will not express clear separation of the pixels as two peaks, in such cases threshold selection become a difficult task. To solve this difficulty several methods have been proposed in the literature [1][2][3][4][5][6]. Otsu [7] proposed discriminant analysis to maximize the separability of the resultant classes. Interaction among the pixels is also a reasonable feature tried in two-dimensional Otsu method, in reference [8]. In entropy based algorithms proposed by Kapur et al. [10] extend the previous work of pun [9] that first uses the concept of entropy for thresholding. This method concludes that when the sum of the background and object entropies reaches its maximum, the threshold value is obtained. In Kapur et al. [10], images which are corrupted with noise or irregular illumination produce multimodal histograms in which a gray level histogram does not guarantee for the optimum threshold selection, because no spatial correlation is considered. In reference [11], Abutaleb extended Kapur's method using two dimensional entropies which are calculated from a two dimensional histogram which was determined by using the gray value of the pixels and the local average of neighborhood gray values of the pixels. As an improvement, later this technique is further simplified by A.D.Brink [12]. Entropy criterion function is applied on 2-D GLSC histogram to select optimum threshold by surpassing difficulties with 1-D histogram by Yang Xiao et al. [13,14]. This work is further extended by Seetharama Prasad et al. [16] using variable similarity measure producing improved GLSC histogram. The ordinary thresholding techniques perform poorly when non-uniform illumination corrupts object characteristics and inherent image vagueness is present. Fuzzy based image thresholding methods have been introduced in the literature to overcome this problem. Fuzzy set theory [5] is used in these methods to handle grayness ambiguity or inherent image vagueness during the process of threshold selection. Fuzzy C-partitions were used on entropic criteria to achieve optimum threshold value by Seetharama Prasad et al. [17]. In reference [15] Type-II fuzzy is used with GLSC histogram with human visual nonlinearity characteristics to identify the optimal similarity measure. Type-II fuzzy sets and a new fuzzyness measure called Ultrafuzziness are introduced by H.R.Tizhoosh [18] and Type-II fuzzy probability partitions methods are applied on GLSC histogram to obtain the threshold by Seetharama Prasad et al. [19]. Ch.V.Narayana et al. [20] used ultra fuzziness and type-II fuzzy sets for automatic image segmentation. In reference [21] Nuno Vieira Lopes et al. introduced fuzzy measures to threshold gray level images with no entropy criterion function to reduce the time complexity for computation and this technique is further automated by Seetharama Prasad et al. [22].

The remaining part of this paper is organized as follows: section 2 describes about some of the existing methods. Section 3 describes the proposed method, section 4 shows comparative results and improved yielding of our method and section 5 ends with conclusion.

1 II.

2 Existing methodologies

Abutaleb [11], Brink [12] did some good work in obtaining the segmentation of a gray image incorporating the spatial correlation among the pixels of the image by constructing the 2D histogram. Fundamentally these methods attracts maximum entropy criterion function to establish the optimal threshold for any given image. There comes another approach by existing method Tizhoosh [18] introduced a new fuzzy membership function along a new fuzzy measure called ultrafuzziness using type II fuzzy sets to compute a threshold for the image segmentation. Our paper basically aims to combine both these techniques to provide a methodology in obtaining a optimal threshold. a) Fuzzy Based Methodology Measures of fuzziness in contrast to fuzzy measures indicate the degree of fuzziness of a fuzzy set. The entropy of a fuzzy set is a measure of the fuzziness of a fuzzy set. The membership degree of any value in the universe of discourse can be estimated by using any fuzzy membership function. Thizhoosh [18] introduced a new fuzzy measure called ultrafuzziness which could replace the use of entropy in threshold calculations.

3 i. Fuzzy S-membership function

To measure the image fuzziness the most used S-membership degree function as shown in Equation (1) which comprises of three unknown quantities a, b and c must be estimated from the image statistics. (2)= ? ? ? ? ? 0 ?? < ?? 2 ? ????? ????? ? 2 ?? ? ?? ? ?? 1 ? 2 ? ????? ????? ? 2 ?? ? ?? ? ?? 1 ?? ? ??**(1)**??=? 1 ?? ? (?? ?? ? ??) 2 ?? ??=1**(3)**

From Equations (**??**) and (3 c = µ + ??**(5)**

ii. Type I fuzzy sets

The most common measure of fuzziness is the linear index of fuzziness. For a MxN image subset A? X with gray levels g ? [0, L-1], the linear index of fuzziness can be estimated as follows?? 1 (A)= 2 ???? ? ?(ð ??"ð ??")× ?????? ???1 ð ??"ð ??"=0 [?? ?? (g), 1-?? ?? (g)]**(7)**

Where ?? ?? (g) is obtained from Equation (1). So the optimal threshold can be obtained though maximizing the linear index of fuzziness criterion function that is given byt * = Arg max { ??(A: T)}, 0 ? T ? L-1**(8)**

iii. Type II fuzzy sets Definition. A tupe II fuzzy set Ã is defined by type II membership function X ?? Ã (??, ??), where x ? X and u ? ?? ?? ? [0,1] Ã can be expressed in the notation of fuzzy set as The interval between lower/left and upper/right membership values (bounded region) will capture the footprint of uncertainty A type II fuzzy set can be defined from type I fuzzy set and assign upper and lower membership degrees to each element to construct the foot print of uncertainty as shown in Figure 3. a more suitable definition for a type II fuzzy set can be given as follows:Ã ={((x, u), ?? Ã (x,u))| ? ? ?? ? X, ? ?? ? ?? ?? ?[0,1]}, in which 0 ? µ Ã (??, ??) ? 1Ã = {??, ?? ?? ? (??), ?? ?? (??)| ? ?? ? ??, ?? ?? (??) ? µ(??) ? , ?? ?? (??), µ ? [0,1]**(9)**

The upper and lower membership degrees ?? ?? and ?? ?? of initial membership function µ can be defined by means of linguistic hedges like dilation and concentration:?? ?? (??) = [µ(??)] ??.ð ??"ð ??" , ?? ?? (??) = [µ(??)] ?? ,

Hence, the upper and lower membership values can be defined as follows:?? ?? (??) = [µ(??)] ?? ?? , ?? ?? (??) = [µ(??)] ?? , Where Î?" ? (1, ?) but Î?">2 is usually not meaningful for image data. iv. Tizhoosh Ultrafuzziness

The degrees of membership is defined without any uncertainty as type I fuzzy sets, automatically the ultrafuzziness also tend to zero. When individual membership values can be indicated as an interval, the amount of ultrafuzziness would increase. The maximum ultrafuzziness is one when the information of membership degree values totally ignored. For a type II fuzzy set, the ultrafuzziness is defined as ? for a M x N image subset Ã? X with gray levels g ? [0, L-1], histogram h(g) and membership function µ Ã (ð ??"ð ??"). The ultrafuzziness of the gray level image is formulated as follows:??(Ã)= 1 ???? ? ?(ð ??"ð ??")?? [???1 ð ??"ð ??"=0 ?? ?? (ð ??"ð ??") ? ?? ?? (ð ??"ð ??")]**(10)**

Where One dimensional histogram based methods does not consider the spatial correlation between pixels in an image. This is simple to implement and does not consider the physical location of pixel and its interaction with neighboring pixels. When different images with an identical histograms, will result in the same threshold value, to avoid this kind of problems spatial methods are employed. In the later approach it involves the local average gray values of the pixels and their probability distribution in making two dimensional entropy based segmentation procedure, resulting in better than its earlier methods. From the literature many researchers worked and made many betterments to the existing methods.?? ?? (ð ??"ð ??") = [µ(ð ??"ð ??")] 1 ?? , ?? ?? (ð ??"ð ??") = [µ(ð ??"ð ??")] ?? , Î?" ? (1, 2)

4 i. Entropy based methods

The gray level of each pixel and the average gray level value of its neighbourhood are examined. A.S. Abutaleb [11] first tried with this approach as the frequency of occurrence of each pair of gray level and local average gray level called bin is computed. For any generalized gray level image having no fuzziness possessed in the image, produces two peaks with one valley corresponds to foreground and background respectively. They can be separated by choosing the threshold that maximizes entropy in the two groups. Later A.D. Brink [12] improved by not considering some portion of the 2D histogram as the off diagonal bins being contributed by edges and noise in the image where as bulk of the histogram, including the peaks, lies on or near the leading diagonal of the 2D histogram.

5 a. Abutaleb's method

Let the gray level be divided into m values and the average gray level also divided into the same m values. At each pixel, the average gray level value of the neighbourhood is calculated. This forms a pair: the pixel gray level and the average of the neighbourhood. Each pair belongs to a dimensional bin. The total number of bins obviously m×m and the total number of pixels to be tested is N×N. The total number of occurrences, ð ??"ð ??" ???? , of pair (x,y) is divided by total number of pixels, ?? 2 , defines the joint probability mass function.

6 ?? ???? =

ð ??"ð ??" ???? ?? 2 x and y = 1, ..., m. The two groups represents object and background O,B, with two different probability mass functions. If the threshold is located at the pair (s, t) then the total area under ?? ???? (x = 1, ?, s and y = 1, ?., t) must equal one. The entropy base function, ?(s, t) = H(O) + H(B) where H(O) is entropy of the object and H(B) entropy of the back ground. This algorithm searches for the values of s and t that maximizes ?(s, t). There the threshold is located. b. Yang Xiao et. al method Yang Xiao et al. [13][14][15] further simplified this approach by defining a GLSC histogram is constructed by considering the similarity in neighborhood pixels with some adaptive threshold value as similarity measure (?).

Let f (x, y) be the gray level intensity of image at (x, y).F= { f (x, y)|x Ñ?"[1,Q], y Ñ?"[1,R]} of size Q x R. The gray level set {0, 1, 2, ... 255} is considered as G throughout this paper for convenience. The image GLSC histogram is computed by taking only image local properties into account as follows. Let g(x ,y) be the similarity count corresponding to pixel of image f (x, y) in N X N neighborhood, where N is any positive odd number in range **??**3, min(Q/2,R/2)]. g(x+1,y+1)= ? ? ? (|ð ??"ð ??"(?? + 1, ?? + 1) ? ð ??"ð ??"(?? + ??, ?? + ??)| ? ?) ???1 ?? =0 ???1 ??=0 (11)

Where, ? (|ð ??"ð ??"(?? + 1, ?? + 1) ? ð ??"ð ??"(??+ ??, ?? + ??)|) = ? 1 ??ð ??"ð ??" |ð ??"ð ??"(?? + 1, ?? + 1) ? ð ??"ð ??"(?? + ??, ?? + ??)| ? ? 0 ???????????????? (**12**)

GLSC histogram is constructed with the correlated probability at different gray level intensities from equations (**??**1) and (12) as follows.

h(k, m)=P(f(x, y)=k and g(x, y)=m) (13) Where, P is the gray level correlation probability computed for all pixels with intensity k Ñ?" G with correlation m Ñ?" {1, 2,..NXN} and histogram is normalized [12]. As author discussed in [11][12] with similarity measure?=4 and N=3. Seetharama Prasad et al. [16] made few changes by taking local and global parameters in deciding the similarity measure ?. Due to the computational penalty of the Otsu method, a statistical parameter Standard Deviation has been adopted to decide the similarity measure for every NXN map, keeping global standard deviation unchanged. Therefore ? is computed as the difference between global standard deviation of the entire image and standard deviation of local NXN map. ? = |std g -std l | From this discussion it is so clear that pixel individual gray value and its positioning in the image are taken into account for entropy computation for background and object, produces optimum threshold where the maximum entropy occurs.

7 III.

8 Proposed method

The optimal threshold is determined by optimizing a suitable criterion function obtained by taking ultrafuzziness into account from the gray level distribution of the image and spatial correlation features of the image.

Let f(x, y) be the gray value of the pixel located at the point (x, y). In For r ?{0, 1, 2, ?., 255}. For the sake of convenience, we denote the set of all gray levels {0, 1, 2, ?., 255} as G.

In order to make use of more information present in the image , construct a index table by this way exploit the spatial correlation that exist among pixels of an image in pursuance of optimal threshold value as two-dimensional histogram serve in 2-D entropic approaches. To construct index table of a given image we proceed as follow. Calculate the average gray value of the immediate neighborhood of each pixel. Let g(x, y) be the average of the neighborhood of the pixel located at the point (x, y). The average gray value for the 3 × 3 neighborhood of each pixel is calculated as g(x, y)=1 9 ? ? ð ??"ð ??"(?? + ??, ?? + ??) 1 ?? =?1 1 ??=?1**(14)**

Now g(x, y) holds the local average value for the corresponding value at f(x, y). for g= 0 to 255k= ? ? ð ??"ð ??"(??, ??) ??????? ð ??"ð ??"(??, ??) = ð ??"ð ??" ???1 ?? =0 ?? ?1 ??=??

Second order statistics yields refined results than the first order statistics. We have employed the second order statistics by considering average of each pixel's averages computed throughout the imagekavg(r)= k/h(r) ; r?{0, 1, ?., 255}**(15)**

Where kavg is a vector holds all relative gray numbers for the corresponding gray number, g. a) Fuzzy concepts Fuzzy membership degree (FMD), ?? ð ??"ð ??" of each gray has been computed using S-fuzzy membership function from equation (1). The upper and lower membership values ?? ?? (ð ??"ð ??") and ?? ?? (ð ??"ð ??") can be generated for type-II fuzzy sets which are useful in ultrafuzzy calculations. As spatial correlation of the image is serving better with entropic approaches the same concept can be exploited towards fuzzy approaches. In this regard the relative gray value is computed from local averages of the gray image with 3×3 map for every gray value. Relative fuzzy membership degree (RFMD) of every gray value is the FMD of its relative gray value to use in ultrafuzzy computation. b) Proposed algorithm 1. Calculate image histogram. 2. For each pixel find the local average with a 3×3 map. 3. For the pair: gray value and its average gray value find the number of occurrences, called a bin. 4. For the each gray value, compute the average gray value of all bins and call it as relative gray value.

9 Construct an index table putting gray values and

their corresponding relative gray values together.

6. Select S-membership function, µ(g). 7. Initialize the position of the membership function. 8. Shift the membership function along the gray-level range and compute fuzzy membership degrees (FMDs). 9. Calculate in each position the upper and lower membership grades ?? ?? (ð ??"ð ??") and ?? ?? (ð ??"ð ??"). 10. RFMD of any gray value is the FMD of its relative gray value sourced from the index table. 11. Compute the ultrafuzziness value for each gray value by substituting its corresponding RFMD, using Equation (10). 12. Find out the ð ??"ð ??" ?????? with minimum ultrafuzziness. 13. Threshold the image with ?? = ð ??"ð ??" ?????? .

10 IV.

11 Results and discussions

To illustrate the performance of the proposed methodology we consider 14 images as an image set having similar and dissimilar gray level histogram characteristics, varying from uni-model to multimodal. Gold standard groundtruth images are generated manually to measure a parameter efficiency (?) based on misclassification error [3] and Jaccard Index [4]. This ? would be 0 for absolutely dissimilar and 100 for exactly similar image as result. Figure 4 shows original image set and their possible gold standard threshold image set. From the experiments for each image we obtain misclassification error values against its corresponding ground truth image from different methods including Otsu's, H.R.Tizhoosh and Proposed in Table 1.

Dataset Ground truth Otsu Tizhoosh Proposed House

(D D D D) F b) Jaccard Index

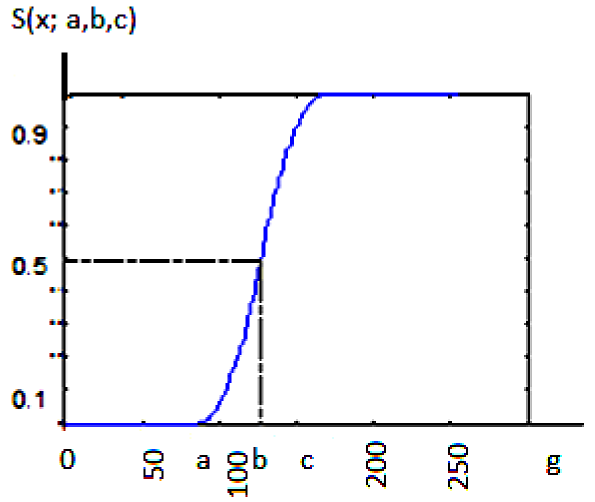
The another similarity measure is the Jaccard Index [4] known as Jaccard similarity coefficient, very popular and frequently used as similarity indices for binary data. The area of overlap A j is calculated between the binary image B j and its If the thresholded object and corresponding gold standard image G j (associated ground truth image) are exactly similar then the measure is 100 and the measure 0 represents they are totally dissimilar, however the higher measure indicates more similarity. Table 2 represents the effectiveness of the proposed method, and Figure 6 shows the superiority of the proposed method against Otsu and Tizhoosh methods. The proposed method has highest average performance of 93.34% with the lowest standard deviation 5.47% when evaluated with Jaccard Index as listed in TABLE 2. In contrast Otsu algorithm with 76.88% average performance and 26.96% standard deviation and Tizhoosh method average performance of 79.97% and 29.02% standard deviation. Hence the proposed method is clearly showing much better performance over existing methods. From the experiments for each image we obtain ?? % for Otsu, Tizhoosh and proposed methods as shown in TABLE 1. These values are compared with assumed gold standard image data. Figure 5 confirms a variation in above said methods on histogram range for image set considered against Otsu method. Efficiency (?) is calculated for each technique on image set with Equation (16). A mean (?) and standard deviation (?) are calculated on efficiency in order to show the effectiveness of the proposed and other methods as in TABLE 1. A mean 96.48 and standard deviation 2.97 is obtained from the proposed method which confirms the qualitative improvement over the existing methods.

12 Conclusion

In this paper a new methodology for segmentation based on spatial correlation in gray image and ultrafuzziness of type-II fuzzy sets is addressed. To decide the fuzzy membership degree we have introduced a new concept called relative gray value and its corresponding relative fuzzy membership degree which is computed from a novel approach employed with two dimensional histogram. This is performing quite good on many complex images over standard methods in the literature. We tried Otsu and Tizhoosh ultrafuzzy methods to compare the results with our method. However, this method can be further improved in the lines of spatial correlation features where some alternate approaches using 5×5, 7×7 or any higher order local maps. Our method is effectively working on low contrast images whose objects are not clearly distinguished from background. Efficiency of threshold selection is demonstrated with experimental results. We assume a reasonable contrast enhancement for low contrast images. Performance evolution is carried out with the help of two popular approaches; Misclassification error and Jaccard Index on the proposed work. [1]

1

Figure 1: Fig. 1 :



21

Figure 2: Fig. 2 :Year 1 ???

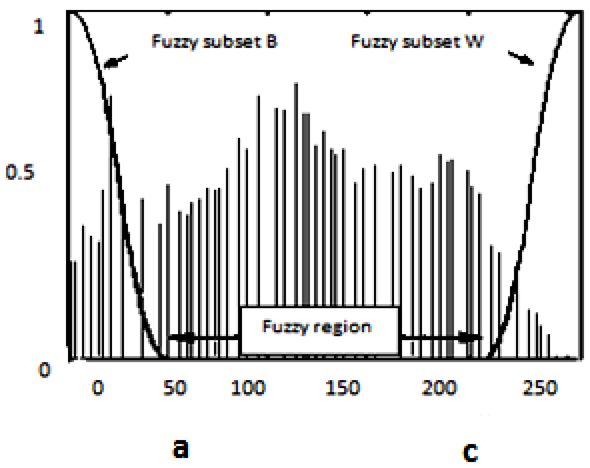
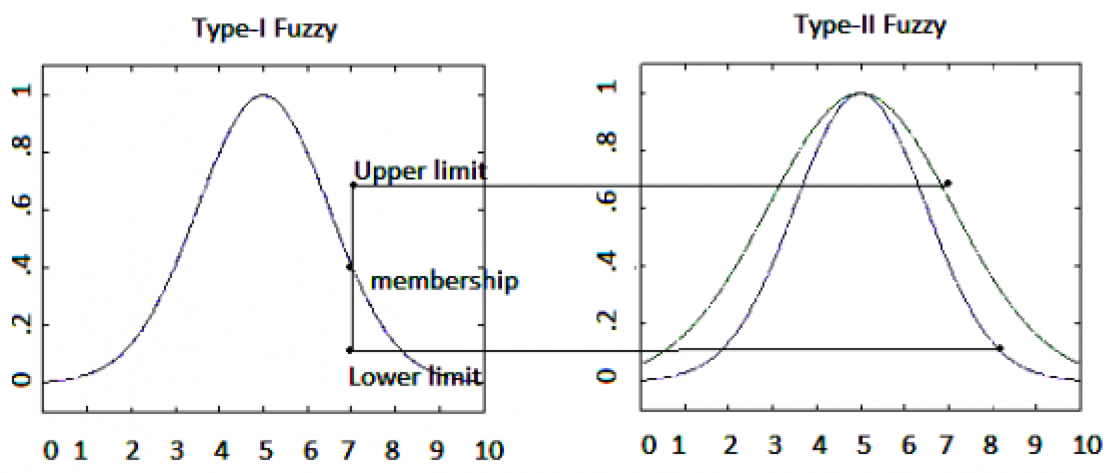


Figure 3:



3

Figure 4: Fig. 3 :

Figure 5:

Figure 6:

4

Figure 7: Fig. 4 :

5

Figure 8: Fig. 5 :

6

Figure 9: Fig. 6 :

1

Sl. no	Image	Otsu	Tizhoosh Proposed	
1	House	99.64	98.48	98.48
2	Wheel	98.58	99.27	97.44
3	Trees	30.89	27.42	99.86
4	Blood	96.95	98.53	95.5
5	3Blocks	93.46	87.16	92.56
6	Potatoes	98.79	97.15	96.83
7	Rhino	92.92	96.97	93.69
8	Coins	96.17	98.61	97.84
9	Stones	99.42	99.49	99.06
10	Pepper	90.28	85.96	99.68
11	Fleck	77.44	99.03	94.99
12	X-image	66.85	95.69	94.64
13	Pattern	74.51	51.33	100
14	Animal	50.99	49.99	99.53
MEAN (?)		83.93	85.32	96.48
STD (?)		21.38	23.78	2.97

Figure 10: Table 1 :

2

Sl.no	Image	Otsu	Tizhoosh	Proposed
1	House	99.23	97.01	97.01
2	Wheel	97.19	98.55	95.01
3	Trees	18.27	15.89	99.72
4	Blood	94.08	97.10	91.43
5	3Blocks	87.73	77.24	86.14
6	Potatoes	97.61	94.47	93.89
7	Rhino	86.78	94.12	88.13
8	Coins	92.62	97.27	95.77
9	Stones	98.85	99.00	98.14
10	Pepper	96.88	91.27	82.28
11	Fleck	63.18	98.09	90.46
12	X-image	50.22	91.75	89.83
13	Pattern	59.37	34.53	100
14	Animal	34.22	33.32	99.07
	MEAN (?)	76.88	79.97	93.34
	STD (?)	26.96	29.02	5.47

Figure 11: Table 2 :

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