Maximum Likelihood Criterion Based On Polygon Inequality

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***Abstract*--This letter proposes a maximum likelihood criterion for soft-decision decoding algorithm of binary linear block codes over AWGN channel based on natural polygon inequality. In that criterion, Hamming weights are computed instead of Euclidean distances. As a result, decoding complexity is reduced in both generating test patterns and computing Euclidean distances.**

Keywords- polygon inequality, soft-decision decoding, Hamming weight, Euclidean distance.

I. INTRODUCTION

Basically, error control coding with forward error-correction can be categorized into hard decision decoding (HDD) and soft decision decoding (SDD). HDD algorithms have the advantage of low decoding complexity but the disadvantage of low error-correction performance. On the contrary, SDD algorithms have the advantage of better error-correction performance by two to three dB coding gain with respect to that of HDD algorithms [1], but the disadvantage of high decoding complexity. For better error-correction performance, many studies proposed some SDD algorithms based on such as the threshold of Eb/No [2], stop criterion [3], reliabilities [4], voting [5] or for a particular code [6]-[7] to reduce decoding complexity with slight or without degradation in error-correcting performance. To reduce decoding complexity, this letter proposes a soft-decision maximum likelihood criterion for SDD algorithm by computing Hamming weights (HWs) rather than Euclidean distances (EDs). If the criterion is satisfied, the code word is the maximum likelihood (ML) one. Hence, generating test patterns (TPs) is unnecessary. Certainly, computing EDs is not required. Thus, decoding complexity is reduced. The higher the SNR is, the more chance the code word satisfies the criterion. In the next section, The Chase algorithm [8] is introduced first and the decoding algorithm with the proposed criterion is presented. In addition, the criterion is defined and proved as well. Following that, simulation results show that the decoding complexity with the help of proposed criterion is reduced. In turn, the strength and the weakness of decoding algorithm with the proposed criterion are summarized in the conclusion.

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II. DECODING ALGORITHM WITH THE PROPOSED CRITERION

Assume that C(n,k,dmin) denotes a code with code length n, message length k, minimum Hamming distances dmin between code words, and number of error-correcting capability $t=\left\lfloor \frac{d_{\min}-1}{2} \right\rfloor$. And further, algorithm with the proposed criterion is termed as natural polygon inequality (NPI) based SDD algorithm. Now, consider a code word $C=[c_1\ c_2\ \ldots\ c_n]$ is transmitted using BPSK modulation, where c_i denoted the ith bit of C. And suppose that the received signal at the output of demodulator is given as $Y=[y_1\ y_2\ \ldots\ yn]$. Let $A=[\alpha_1\ \alpha_2\ \ldots\ \alpha_n]$, which is absolute value |Y|, be the reliability word of Y. In addition, R is the word received from the output of hard decision (HD) with Y as the input; that is R=HD(Y), where HD given as (1).

$$HD(y_i)=\begin{cases}0, & y_i \le 0\\ 1, & y_i > 0\end{cases} \qquad (1)$$

With availability of R and A, the decoding steps of Chase algorithm are as follows:

Step 1. Do HDD to R

$$C_R=HDD(R) \qquad (2)$$

Step 2. Do C_R exclusive-OR operation with R as in (3) and then error pattern (EP_R) is found.

$$EP_R= C_R \oplus R \qquad (3)$$

If there is no error, the decoding process finishes with CR. If not, continue the following steps.

Step 3. Sort the reliabilities $A=[\alpha_1\ \alpha_2\ \ldots\ \alpha_n]$ in ascending order and which gives a new reliability vector $A_S= [\alpha_{S1}\ \alpha_{S2}\ \ldots\ \alpha_{Sn}]$, where $\alpha_{Si} < \alpha_{Sj}$ for $i < j$. Also, a position vector $P_S=[p_{S1}\ p_{S2}\ \ldots\ p_{Sn}]$ keeps the original position of $[\alpha_{S1}\ \alpha_{S2}\ \ldots\ \alpha_{Sn}]$.

Step 4. By use of Ps, Chase algorithm generates a number of TP according to the positions of bits with least reliabilities in R.

Step 5. Do R exclusive-or operations with TPs as given in (4),

$$Zs=R \oplus TPs \qquad (4)$$

Step 6. Do HDD to words Zs as given in (5).

CCs=HDD(Zs) (5)

Step 7. Find out the code word in the code word candidates (CCs) for which has the minimum ED from R.

The decoding process of NPI algorithm is the same as that of Chase except with one more step after step3 by testing (6). Utilizing the position word P_S and programming skill of indirect addressing, C_R is a ML code word if (6) satisfied.

$$\sum_{i=1}^{d_{\min}-W\{EP_{R_S}\}}(EP_{R_S}) = W\{EP_{R_S}\} \quad (6)$$

where

EP_{R_S} : sorted EP_R in ascending order according to P_S.

$W\{EP_{R_S}\}$:Hamming weights of EP_{R_S} .

Equation (6) implies that the error bits are included in the least reliable bits. If (6) holds for the associated C_R, decoding is completed. Hence, generating TPs and computing EDs are unnecessary from step4 to step7 for the NPI algorithm compared to that of Chase algorithm. If EP_{R_S} does not satisfy (6), NPI algorithm follows from step4 to the last step of Chase algorithm to check if C_R is a ML code word.

Before proving that (6) determines the associated C_R a ML code word, a theorem of natural polygon inequality is introduced based on the theorem of triangle inequality.

Theorem- Natural polygon inequality (NPI)

Suppose that a polygon is formed with M edges, where the length of edges $E_1 \leqq E_2 \leqq \ldots \leqq E_M$ and $E_M < E_1+E_2+\ldots+E_{M-1}$. Then, the summation of edges' length has following natural inequality (7) and which is termed as the NPI.

$$\sum_{i=1}^{\left\lfloor \frac{M}{2} \right\rfloor} E_i \le \sum_{i=\left\lfloor \frac{M}{2} \right\rfloor+1}^{M} E_i \quad (7)$$

Now, we prove that (6) determines the associated C_R a ML code word.

Proof- Since $A_S=[\alpha_{S1}\ \alpha_{S2}\ \alpha_{S3}\ \ldots\alpha_{Sn}]$ is the sorted reliability and if (6) is satisfied, the maximum ED_{C_R} of code word C_R from the received word R is

$$ED_{C_R} \le \sum_{i=1}^{d_{\min}-W\{EP_{R_S}\}} \alpha_{Si}$$

Moreover, any other code word C_X with minimum ED_{C_X} from the received word R is

$$ED_{C_x} \ge \sum_{i=d_{\min}-W\{EP_{R_S}\}+1}^{d_{\min}} \alpha_{Si}$$

Referring to (7), we observed that

$$ED_{C_R} \le ED_{C_X}$$

Hence, C_R is a ML code word. The following example helps to clarify the proof above.

Example: Assume that Golay (23,12,7) is used and C_R satisfies (6). Then referring to (6), the maximum value of ED_{C_R} is

$ED_{C_R} \le \alpha_{S4} + \alpha_{S5}$, in case of $W\{EP_{R_S}\}$ =2.And any other code word CX with the minimum value of ED_{C_x} is

$$ED_{C_x} \ge \alpha_{S1}+\alpha_{S2}+\alpha_{S3}+\alpha_{S6}+\alpha_{S7}$$

when $W\{EP_{R_S}\}$ =2

Since $A_S=[\alpha_{S1}\ \alpha_{S2}\ \alpha_{S3}\ \ldots\alpha_{Sn}]$ is the sorted reliability in ascending order, the reliabilities inequality $\alpha_{S4}+\alpha_{S5} \le \alpha_{S6}+\alpha_{S7}+\alpha_{S3}+\alpha_{S2}+\alpha_{S1}$ naturally holds. Intuitively, ED_{C_R} is less than ED_{C_X} . CR is a ML code word.

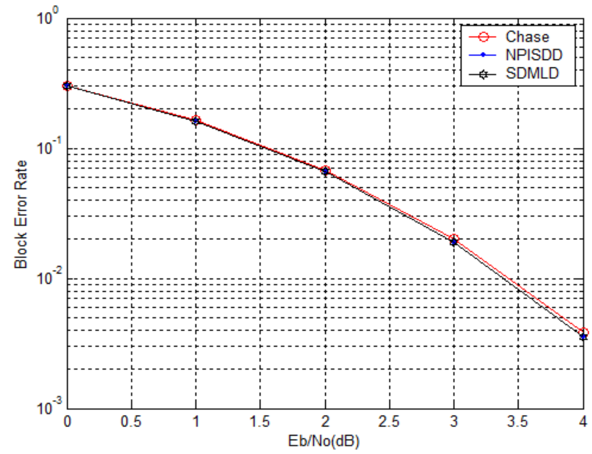

Fig.1. Error-correcting performances

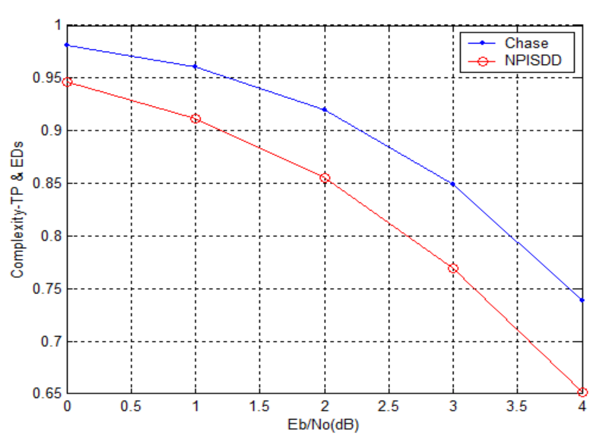

Fig.2. Comparison of decoding complexity in terms of TPs and EDs

III. Simulation results

As shown in Fig. 1, the proposed NPI algorithm and the Chase algorithm has almost the same error-correcting performance as the soft-decision ML decoding. In addition, the decoding complexity in generating TPs and computing EDs is reduced by around from 4% to 9% for the proposed NPI algorithm from 0 dB to 4 dB of Eb/No compared to that of Chase algorithm. In other words, around 4% to 9% in the decoding process is computing HWs in stead of EDs.

IV. Conclusion

We summarize the advantage and disadvantage of NPI algorithm by making a comparison with those of Chase algorithm. Obviously, the NPI algorithm has the advantage of less decoding complexity in generating TPs and computing EDs by around 9% at 4 dB of Eb/No as shown in Fig 2. However, this is a trade-off of one more step of testing with (6). Nevertheless, (6) is a soft-decision maximum likelihood criterion without computing EDs and which is the innovative point in this letter.

V. References

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