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On Baysian Estimation of Loss of Estimators of Unknown Parameter of Binomial Distribution By Randhir Singh

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Abstract

This paper aims at the Bayesian estimation for the loss and risk functions of the unknown parameter of the binomial distribution under the loss function which is different from that given by Rukhin(1988). The estimation involves beta distribution, a natural conjugate prior density function for the unknown parameter. Estimators obtained are conservatively biased and have finite frequentist risk.

Index terms— Bayes Estimator, Loss Function, Risk Function, Binomial Distribution.

1 Issue ersion I

V IV (F) Rukhin(1988) introduced a loss function given by, L(?, ?, ?) = w(?, ?)? ? 1 2 + ? 1 2 (1.1)

Where,? is an estimator of the loss function w(?, ?) ,which is non-negative. Guobing(2016) used this loss function and derived estimates of the loss and risk function of the parameter of Maxwell's distribution. Singh (2021) took various forms of w(?, ?) and derived estimates of the loss and risk function of the parameter of a continuous distribution which gives Half-normal distribution,Rayleigh distribution and Maxwell's distribution as particular cases. Rukhin(1988) considered the Bayesian estimation of the unknown parameter ? of the binomial distribution by takingw(?, ?) = (? ? ?) 2 (1.2)

In this paper,Bayes estimate of the unknown parameter ? of the binomial distribution has been obtained by replacing w(?, ?) by w 1 (?, ?) given byw 1 (?, ?) = h(?)(? ? ?) 2 (1.3)

Where,h(?) = 1 {?(1 ? ?)} (1.4)

2 Notes

Summary-This paper aims at the Bayesian estimation for the loss and risk functions of the unknown parameter of the binomial distribution under the loss function which is different from that given by Rukhin(1988). The estimation involves beta distribution, a natural conjugate prior density function for the unknown parameter. Estimators obtained are conservatively biased and have finite frequentist risk. Let the random variable X follows binomial distribution with parameters n and ?.Where ? is unknown satisfying 0 ? ? ? 1.The prior p.d.f of ?,denoted by ? 1 (?) is as follows:

3 Global

? 1 (?) = ? ??1 (1??) ??1 B(?,?) if ? ? 0,? ? 0,0 < ? < 1 0 Otherwise (2.1)

Under the assumption of prior probability density function (p.d.f.) for ? as above,Bayes estimates of ? derived by Rukhin (1988) were as follows:For ? ? 0, ? ? 0 ? B (X) = (X + ?) (n + ? + ?) (2.2) ? B (X) = (X + ?)(n + ? ? X) (n + ? + ?) 2 (n + ? + ? + 1) (2.3)

and for ? = 0, ? = 0? 0 (X) = X n (2.4) ? 0 (X) = X(n ? X) n 2 (n + 1) (2.5)

It was shown thatE ? L(?, ? 0 , ? 0) = ? (2.6)

Under,w 1 (?, ?) as above, the corresponding Bayes estimate is given by, For ? ? 0, ? ? 0? 1B (X) = E{?h(?)/X} E{h(?)/X} (2.7) Or, ? 1B (X) = (X + ? ? 1) A ? 2 (2.8)

On simplification,provided,A = n + ? + ? > 2 and,? 1B (X) = E{?h(?)/X} ? {? 1B (X)} 2 E{h(?)/X} (2.9)

4 Notes

Estimation of Loss and Risk of the Parameter of Binomial Distribution II.

5 Issue ersion I V IV (F)

On Baysian Estimation of Loss of Estimators of Unknown Parameter of Binomial DistributionE ? L(?, ? 1B , ? 1B) = E ? [h(?)(? ? (X + ? ? 1)(A ? 2) ?1) 2](A ? 2) 1/2 + (A ? 2) ?1/2 (2.11)

Or,E ? L(?, ? 1B , ? 1B) = [n+h(?)(1 ? ? + ?(? + ? ? 2)) 2](A?2) ?3/2 +(A?2) ?1/2 < ? (2.12) In this case, R(?, ? 1B) = E ? {h(?)(? ? ? 1B)} 2 (2.13) Or, R(?, ? 1B) = [n + h(?){1 ? ? + ?(? + ? ? 2)} 2](A ? 2) ?2 (2.14)

As mentioned by Keifer (1977),an estimator ?(X)is said to be conservatively biased if,E ? {?(X)} ? R(?, ?) = E ? {w(?, ?)} (2.15)

In the light of this condition,? 0 (X) as given by Rukhin (1988) is not conservatively biased. In this case,E ? {? 1B (X)} = 1 A ? 2 (2.16)

Let ? 0B (X) and ? 0B (X)be values of ? 1B (X) and ? 1B (X) ,respectively when,? = ? = 0.If possible let ,E ? {? 0B (X)} ? R(?, ? 0B (2.17) which holds if, ?2? 2 + 2? ? 1 ? 0 (2.18)

which is a contradiction,since 0 < ? < 1 and maximum value of ?2? 2 + 2? ? 1 is? 1 2 which corresponds to ? = 1 2 .Moreover,?2? 2 + 2? ? 1 = ?1 for ? = 1 and ? = 0 Thus,? 0B (X) is not conservatively biased.

When ? = ? = 1,we have,E ? {? 1B (X)} = R(?, ? 1B) = 1 n (2.19)) Or, ? 1B (X) = 1 A ? 2 (2.10) on simplification,provided,A = n + ? + ? > 2.

We,see that, in this case ? 1B (X) does not depend upon X and is function of n,? and ?

6 Notes

Issue ersion I V IV (F)

On Baysian Estimation of Loss of Estimators of Unknown Parameter of Binomial Distribution g(?) is a monotonically increasing function of ? over the set S = (0, 1) ? {0.5}.Hence, ? 1B (X) as above,presents a valid 'frequentist report' as mentioned by Berger(1985).

The results are summerized in the following: THEOREM.Let (? 1B , ? 1B) be Bayes estimators of the unknown parameter ? of the binomial distribution under the loss function L(?, ?, ?) =1 {?(1??)} (? ??) 2 ? ? 1 2 +? 1 2

and beta prior density with known parameters ? and ?.Then,the frequentist risk E ? L(?, ? 1B , ? 1B) is finite for all values of ? and ? provided 0 < ? < 1.For ? = ? = 0, ? 1B (X) is not conservatively biased. The estimator ? 1B (X) is conservatively biased for? = ? = 1 and for ? = ? > 1 satisfying ? ? 1 + 2?(1??) (2??1) 2 ,? = 0.5.However, if ? = ? > 1, ? = 0.5, ? 1B (X) is also conservatively biased.

When,? = ? > 1, ? = 0. [1]

Figure 1:

5,

which holds if E ? {? 1B (X)} ? R(?, ? 1B) (2.21) Notes

? ? 1 + g(?) (2.22)

.Where,

g(?) = 2?(1 ? ?) (2? ? 1) 2 (2.23)

Figure 2:

6 NOTES

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