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Mathematical and Computer Modeling of the State of Complex Systems under the Influence of Potential Forces

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Abstract

This article considers the problem of determining critical points and areas in a system that is exposed to external forces. As a result, the system can lose its stability and go into a non-equilibrium state, and then collapse and cause various kinds of catastrophes. The study of the problem of identification and prediction of disasters is relevant, because, allows you to take preventive measures to prevent them and reduce the risks of various negative scenarios.

Index terms— catastrophe, deformation, collapse, bifurcation, deformations, system stability.

1 I. Introduction

An analysis of the stability of a certain system is an urgent practical problem, since its reaction even to a small perturbation of the parameters can be so strong that it will lead the system to a catastrophic state of destruction. Natural disasters and other emergencies of a natural and man-made nature are threats to the national security of the country, since their onset leads to significant material damage to the economy and loss of human lives [1].

A catastrophe is the transition of a system from a stable state with small fluctuations and damped oscillations to a state in which the amplitude of these oscillations grows and transfers the system to a new non-equilibrium state.

Methods of catastrophe theory offer tools for studying abrupt and abrupt transitions to a nonequilibrium state of a system as a result of changes in its parameters, i.e. bifurcations. The identification of the main types of bifurcations and the construction of bifurcation diagrams will make it possible to control the parameters of a dynamic system in order to increase its stability.

The mathematical theory of catastrophes arose thanks to the efforts of many scientists, is based on the theory of stability and bifurcations of dynamical systems and analyzes the critical points of a potential function.

The works of the American mathematician G. Whitney laid the foundations for the theory of singularities or singularities at points where the mathematical function is not defined or has irregular behavior [2].

The terms "catastrophe" and "catastrophe theory" were coined by the British mathematician Christopher Zieman and René Thom in the late 1960s and early 1970 s. E.K. Zieman proposed to use the term "catastrophe theory" to combine the theory of singularities, the theory of bifurcations and their applications. Singularity theory provides information about critical points for studying the onset of a "catastrophe", that is, a jump-like transition of a system from one state to another when its parameters change [3].

In his works, R. Tom gives a deep classification of seven fundamental types of catastrophes and analyzes the critical points at which the potential function loses its stable equilibrium [4].

V. I. Arnold expands the "ADE-classification" of catastrophe models, using deep connections with the theory of Lie algebra [5].

The works of the American scientist Gilmore R. (Gilmore R.) is devoted to the practical application of the theory of catastrophes in such areas of science and technology as mechanics, construction, climatology and others [6].

The issues of modeling natural disasters are considered in the work of scientists S.L. Castillo Daza and F. Naranjo Mayorga [7].

3 III. MATHEMATICAL MODEL AND PROBLEM SOLVING METHODOLOGY

An important task in science and technology is to ensure the strength and reliability of industrial and civil facilities in the face of man-made and natural disasters. The task is complicated by the fact that both catastrophes can have a simultaneous impact on an engineering structure. The reason for such synergy may be their mutual influence, since natural disasters can be the primary source of a man-made disaster, and vice versa.

Deformations can lead to the loss of stability of the technical structure and to its collapse, and thus cause a man-made disaster. Reducing the negative consequences of their occurrence depends on how quickly it is possible to predict them, and then effectively use the mechanisms and tools to neutralize the negative consequences.

2 II. Statement of the Problem and Mathematical Model a) Formulation of the Problem

Consider the solution of the problem of the state of an underground engineering structure in a rock mass, which is affected not only by ordinary (classical) stresses, but also by couple stresses (Cosserat Wednesday Cosserat). In this case, classical stresses lead to tensile-compressive and shear deformations, while couple stresses cause deformations of curvature and rotation in the system.

The state of an elastic rock mass with an engineering structure is modeled by an infinite isotropic plane with moment stresses, in which there is a hole of arbitrary shape. It is required to obtain analytical formulas for determining the components of the stress, displacement and rotation functions of the elements of the rock mass around the engineering structure in order to identify critical areas. This complex task will consist of two stages.

First, the state of the main (zero) untouched massif is determined, which is modeled by a solid plane.

Secondly, additional components of the functions of stresses, displacements and rotation of the array, in which there is a hole of arbitrary non-circular shape, are determined.

The stress state of an elastic plane with a hole consists of the components of the main stresses of the untouched massif and the components of additional stresses due to the presence of the hole. Stress functions can be represented as a sum (1): $(\sigma_{xx}, \sigma_{yy}, \sigma_{xy}, \tau_{xx}, \tau_{yy}, \tau_{xy}, \sigma_{xx}^c, \sigma_{yy}^c, \sigma_{xy}^c, \tau_{xx}^c, \tau_{yy}^c, \tau_{xy}^c) = (\sigma_{xx}^0, \sigma_{yy}^0, \sigma_{xy}^0, \tau_{xx}^0, \tau_{yy}^0, \tau_{xy}^0) + (\sigma_{xx}^c, \sigma_{yy}^c, \sigma_{xy}^c, \tau_{xx}^c, \tau_{yy}^c, \tau_{xy}^c)$

where $(\sigma_{xx}^0, \sigma_{yy}^0, \sigma_{xy}^0, \tau_{xx}^0, \tau_{yy}^0, \tau_{xy}^0)$ are the components of the main stresses of the untouched massif;

$(\sigma_{xx}^c, \sigma_{yy}^c, \sigma_{xy}^c, \tau_{xx}^c, \tau_{yy}^c, \tau_{xy}^c)$ are the components of the additional stresses due to the presence of the hole.

where Δ - Laplace operator, value Δ^2 - is the square of the modulus of the wave vector, $\Delta^2 = -k^2$, $\Delta^2 = -k^2$

Kolosov-Muskhelishvili stress functions for rock massif [8].

Let us write in the Cartesian coordinate system the basic (zero) equations of the planar moment theory of elasticity in the case of the absence of bulk ordinary and moment forces in an untouched massif: -equilibrium equations according to the formula (3): $\sigma_{xx} + \sigma_{yy} = 0$; $\sigma_{xx} - \sigma_{yy} = 0$; $\sigma_{xy} = 0$; $\tau_{xx} + \tau_{yy} = 0$; $\tau_{xx} - \tau_{yy} = 0$; $\tau_{xy} = 0$

-Hooke's law for a medium with couple stresses according to formula (4): $\sigma_{xx} = \frac{1}{2}(\sigma_{xx}^0 + \sigma_{xx}^c) + \frac{1}{2}(\sigma_{yy}^0 + \sigma_{yy}^c)$; $\sigma_{yy} = \frac{1}{2}(\sigma_{xx}^0 + \sigma_{xx}^c) + \frac{1}{2}(\sigma_{yy}^0 + \sigma_{yy}^c)$; $\sigma_{xy} = \frac{1}{2}(\sigma_{xx}^0 - \sigma_{xx}^c) + \frac{1}{2}(\sigma_{yy}^0 - \sigma_{yy}^c)$; $\tau_{xx} = \frac{1}{2}(\sigma_{xx}^0 - \sigma_{xx}^c) + \frac{1}{2}(\sigma_{yy}^0 - \sigma_{yy}^c)$; $\tau_{yy} = \frac{1}{2}(\sigma_{xx}^0 - \sigma_{xx}^c) + \frac{1}{2}(\sigma_{yy}^0 - \sigma_{yy}^c)$; $\tau_{xy} = \frac{1}{2}(\sigma_{xx}^0 - \sigma_{xx}^c) + \frac{1}{2}(\sigma_{yy}^0 - \sigma_{yy}^c)$

where the quantities $(\sigma_{xx}^0, \sigma_{yy}^0, \sigma_{xy}^0, \tau_{xx}^0, \tau_{yy}^0, \tau_{xy}^0)$ are the components of the main stresses of the untouched massif;

$(\sigma_{xx}^c, \sigma_{yy}^c, \sigma_{xy}^c, \tau_{xx}^c, \tau_{yy}^c, \tau_{xy}^c)$ are the components of the additional stresses due to the presence of the hole.

components of the main deformations from ordinary stresses; $\epsilon_{xx}, \epsilon_{yy}, \epsilon_{xy}, \kappa_{xx}, \kappa_{yy}, \kappa_{xy}$

-components of the main deformations (curvature) from moment stresses in an intact massif; ν - Young's modulus, G - Poisson's ratio.

The deformations and the component of the rotation vector are related to the components of the displacement vector by formulas (5): $u = \frac{1}{2}(\sigma_{xx}^0 + \sigma_{xx}^c) + \frac{1}{2}(\sigma_{yy}^0 + \sigma_{yy}^c)$; $v = \frac{1}{2}(\sigma_{xx}^0 - \sigma_{xx}^c) + \frac{1}{2}(\sigma_{yy}^0 - \sigma_{yy}^c)$; $\theta = \frac{1}{2}(\sigma_{xx}^0 - \sigma_{xx}^c) + \frac{1}{2}(\sigma_{yy}^0 - \sigma_{yy}^c)$; $\kappa_{xx} = \frac{1}{2}(\sigma_{xx}^0 - \sigma_{xx}^c) + \frac{1}{2}(\sigma_{yy}^0 - \sigma_{yy}^c)$; $\kappa_{yy} = \frac{1}{2}(\sigma_{xx}^0 - \sigma_{xx}^c) + \frac{1}{2}(\sigma_{yy}^0 - \sigma_{yy}^c)$; $\kappa_{xy} = \frac{1}{2}(\sigma_{xx}^0 - \sigma_{xx}^c) + \frac{1}{2}(\sigma_{yy}^0 - \sigma_{yy}^c)$

3 III. Mathematical Model and Problem Solving Methodology

To solve the problem, analytical and numerical methods are used, as well as calculations using the Fortran programming language. The choice in favor of this particular programming language is dictated by the convenience and obtaining a more accurate solution of differential equations.

Let us construct some domains S and $\bar{S}$ on the plane, which are described by the corresponding analytic functions of the complex variables z and $\bar{z}$. Let us assign to each point z of the region S a certain point $\bar{z}$ of the region $\bar{S}$ using relation (6): $(z, \bar{z}) = (6)$

where $(z, \bar{z})$

is a single-valued analytic function in some domain S on the plane of the complex variable z ; i is the imaginary unit ($i^2 = -1$).

, i is the imaginary unit ($i^2 = -1$).

If the reverse correspondence is also possible, then there is a conformal mapping or transformation of the domain S onto $\bar{S}$, and vice versa [7].

If the function (6) has a simple pole at some point, then the point $z = z_0$ corresponds to the point $\bar{z}$,

and the function (6)

will have the form $(f(z)) = (f(z) + \frac{1}{z - z_0})$

with a holomorphic function $(f(z))$ (c-const). The point z_0

can become a potential bifurcation point or a critical point of the problem.

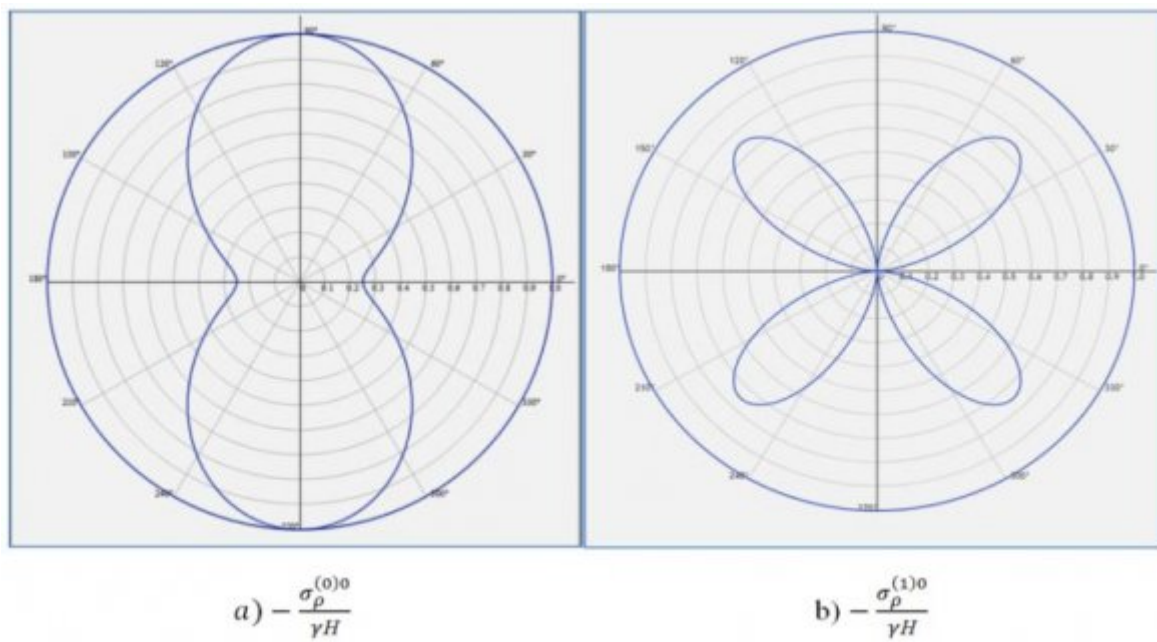
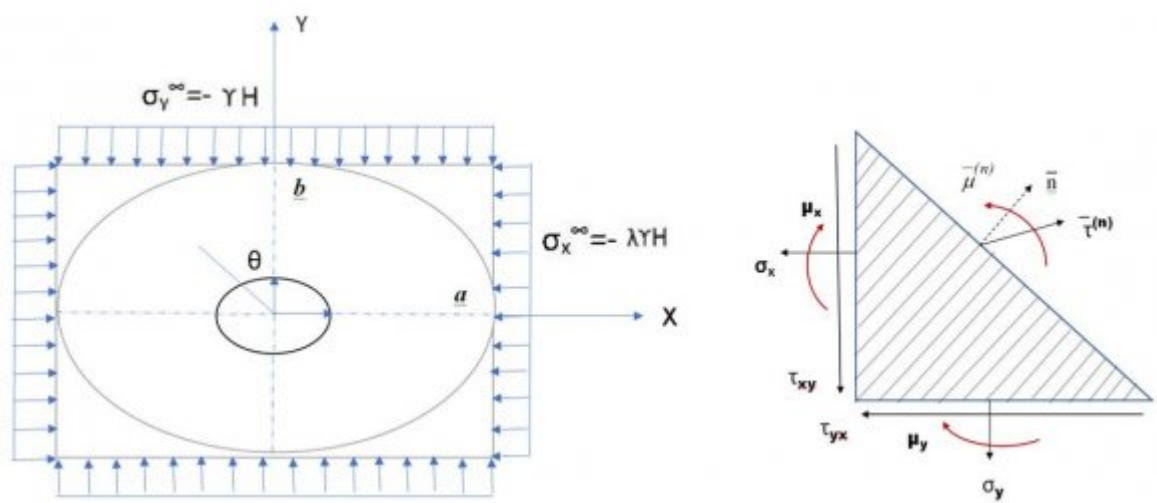


Figure 2: Fig. 1 :

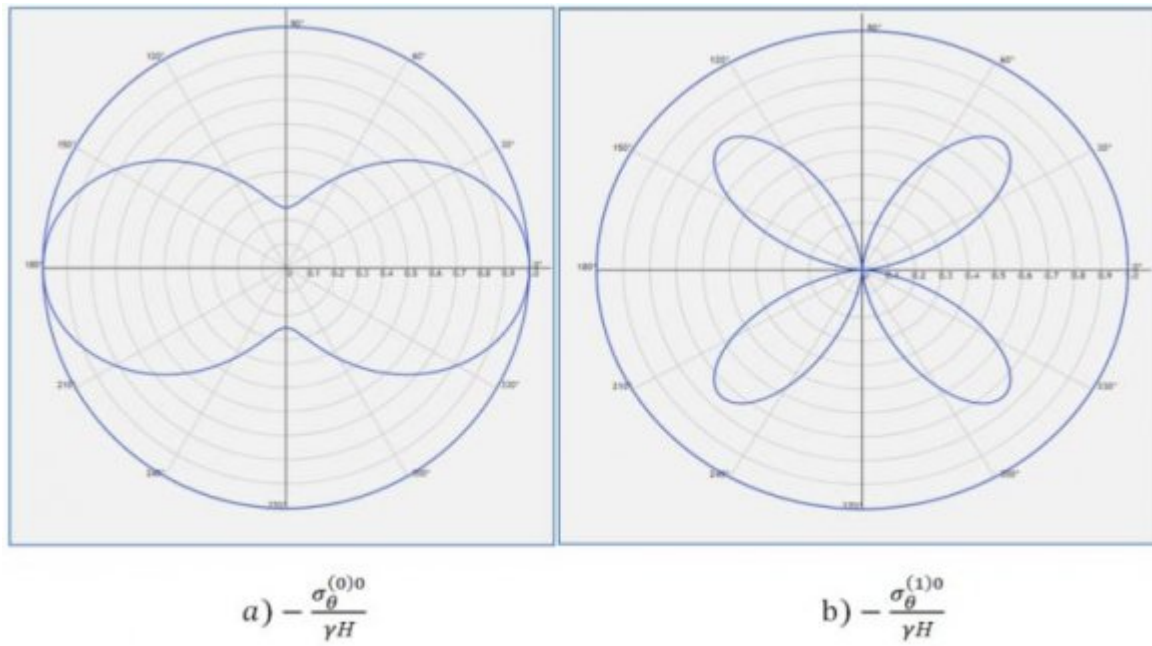


Figure 3:

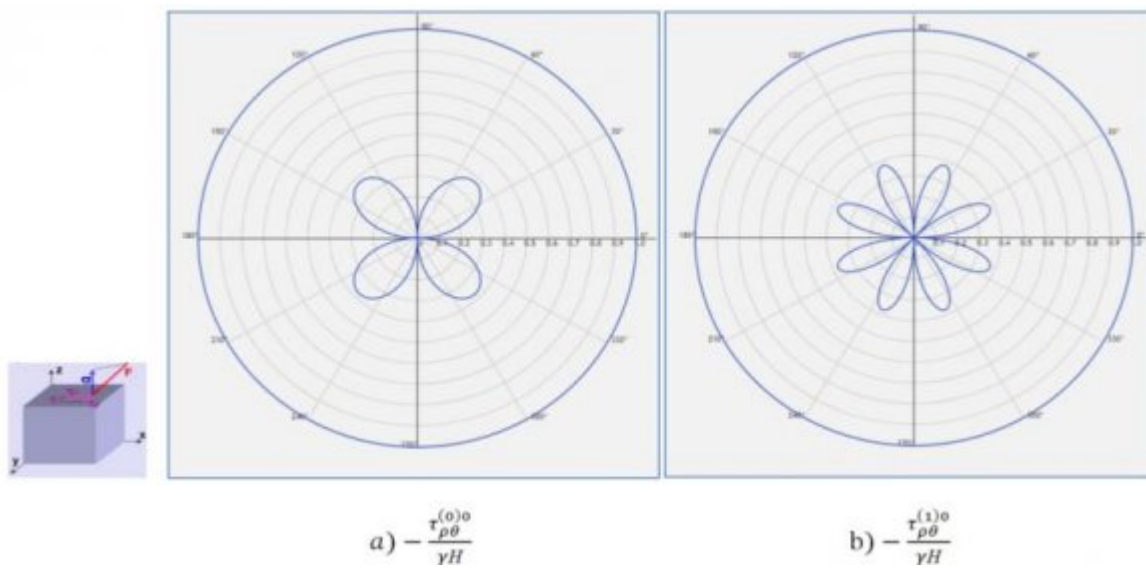


Figure 4:

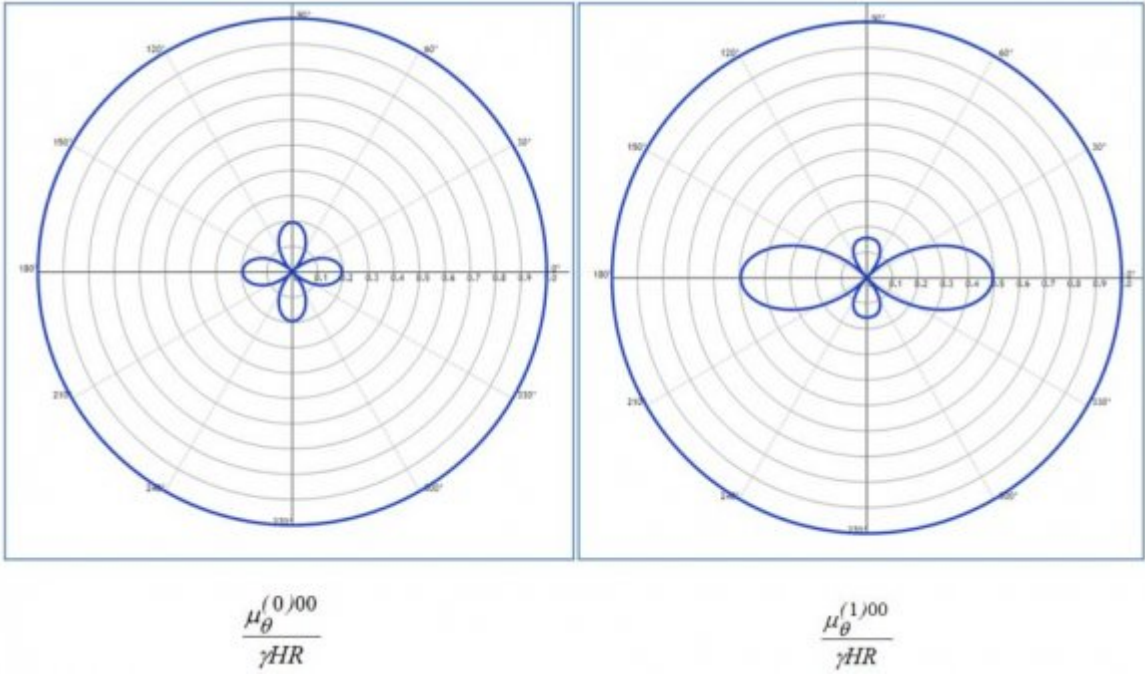


Figure 5:

1

??, degree ?
?? ??

Figure 6: Table 1 :

Table ?? shows the values of additional stresses at the initial moment of time, calculated by formulas (??2) and (24) in dimensionless units. Analysis of Figure ?? shows that the first distribution is symmetrical, while the second distribution is symmetrical about the X and Y axes and asymmetric about the axes of the first and third, as well as the second and fourth quarters.

Total stresses, displacements and rotations for an elliptical working are calculated as the sum of the zero and first approximations (main and additional).

.1 V. Conclusion

Thus, the task of determining the potential stress functions that affect a certain system is completely solved. The advantage of the study is that the task is complicated by taking into account the moment stresses that cause deformations of curvature and rotation in the system. This increases the risk of loss of stability of the system and the rate of its destruction.

The use of such methods as mathematical and computer modeling, the use of numerical methods, contributed to obtaining adequate solutions to the problem.

The graphical implementation of the obtained numerical results makes it possible to see the critical zones in which the system experiences the greatest pressure from external forces. As a result, the system can lose its stability and go into a nonequilibrium state. It is in these areas that engineering construction requires urgent measures such as strengthening and strengthening mechanisms with effective technological solutions to avoid possible catastrophic collapses.

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