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Mathematical and Computer Modeling of the State of Complex Systems under the Influence of Potential Forces

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Kulshat Akanova α, Assem Myrkanova σ, Gaukhar Abdenova ρ & Kenzhebayeva Zhanat ω

Abstract– This article considers the problem of determining critical points and areas in a system that is exposed to external forces. As a result, the system can lose its stability and go into a non-equilibrium state, and then collapse and cause various kinds of catastrophes. The study of the problem of identification and prediction of disasters is relevant, because allows you to take preventive measures to prevent them and reduce the risks of various negative scenarios. The mathematical theory of catastrophes and methods of the theory of stability find practical applications in various fields of applied mathematics, physics, mechanics, biology, as well as in economics and other sciences. The control of the bifurcation parameters of the system, under which the loss of its stability occurs, makes it possible to maintain its equilibrium state and avoid a catastrophe. As an example, the problem of determining the system deformations that arise under the action of the potential function of classical and couple stresses is given. Analytical and numerical methods for solving this problem and performing calculations using the high-level programming language Fortran, which is widely used for scientific and engineering calculations, contribute to obtaining an adequate result.

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I. Introduction

An analysis of the stability of a certain system is an urgent practical problem, since its reaction even to a small perturbation of the parameters can be so strong that it will lead the system to a catastrophic state of destruction. Natural disasters and other emergencies of a natural and man-made nature are threats to the national security of the country, since their onset leads to significant material damage to the economy and loss of human lives [1].

A catastrophe is the transition of a system from a stable state with small fluctuations and damped oscillations to a state in which the amplitude of these oscillations grows and transfers the system to a new non-equilibrium state.

Methods of catastrophe theory offer tools for studying abrupt and abrupt transitions to a non-equilibrium state of a system as a result of changes in its parameters, i.e. bifurcations. The identification of the main types of bifurcations and the construction of bifurcation diagrams will make it possible to control the parameters of a dynamic system in order to increase its stability.

The mathematical theory of catastrophes arose thanks to the efforts of many scientists, is based on the theory of stability and bifurcations of dynamical systems and analyzes the critical points of a potential function.

The works of the American mathematician G. Whitney laid the foundations for the theory of singularities or singularities at points where the mathematical function is not defined or has irregular behavior [2].

The terms "catastrophe" and "catastrophe theory" were coined by the British mathematician Christopher Zieman and René Thom in the late 1960s and early 1970 s. E.K. Zieman proposed to use the term "catastrophe theory" to combine the theory of singularities, the theory of bifurcations and their applications. Singularity theory provides information about critical points for studying the onset of a "catastrophe", that is, a jump-like transition of a system from one state to another when its parameters change [3].

In his works, R. Tom gives a deep classification of seven fundamental types of catastrophes and analyzes the critical points at which the potential function loses its stable equilibrium [4].

V. I. Arnold expands the "ADE-classification" of catastrophe models, using deep connections with the theory of Lie algebra [5].

The works of the American scientist Gilmore R. (Gilmore R.) is devoted to the practical application of the theory of catastrophes in such areas of science and technology as mechanics, construction, climatology and others [6].

The issues of modeling natural disasters are considered in the work of scientists S.L. Castillo Daza and F. Naranjo Mayorga [7].

An important task in science and technology is to ensure the strength and reliability of industrial and civil

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facilities in the face of man-made and natural disasters. The task is complicated by the fact that both catastrophes can have a simultaneous impact on an engineering structure. The reason for such synergy may be their mutual influence, since natural disasters can be the primary source of a man-made disaster, and vice versa.

Deformations can lead to the loss of stability of the technical structure and to its collapse, and thus cause a man-made disaster. Reducing the negative consequences of their occurrence depends on how quickly it is possible to predict them, and then effectively use the mechanisms and tools to neutralize the negative consequences.

II. Statement of the Problem and Mathematical Model

a) *Formulation of the Problem*

Consider the solution of the problem of the state of an underground engineering structure in a rock mass, which is affected not only by ordinary (classical) stresses, but also by couple stresses (Cosserat Wednesday Cosserat). In this case, classical stresses lead to tensile-compressive and shear deformations, while couple stresses cause deformations of curvature and rotation in the system.

The state of an elastic rock mass with an engineering structure is modeled by an infinite isotropic plane with moment stresses, in which there is a hole of arbitrary shape. It is required to obtain analytical formulas for determining the components of the stress, displacement and rotation functions of the elements of the rock mass around the engineering structure in order to identify critical areas.

This complex task will consist of two stages.

First, the state of the main (zero) untouched massif is determined, which is modeled by a solid plane.

Secondly, additional components of the functions of stresses, displacements and rotation of the array, in which there is a hole of arbitrary non-circular shape, are determined.

The stress state of an elastic plane with a hole consists of the components of the main stresses of the untouched massif and the components of additional stresses due to the presence of the hole. Stress functions can be represented as a sum (1):

$$\varphi(\zeta)=\varphi^0(\zeta)+\varphi^{00}(\zeta),\psi(\zeta)=\psi^0(\zeta)+\psi^{00}(\zeta),\ P(\zeta,\bar{\zeta})=P^0(\zeta,\bar{\zeta})+P^{00}(\zeta,\bar{\zeta}) \tag{1}$$

where $\varphi^0(\zeta),\psi^0(\zeta),P^0(\zeta,\bar{\zeta})$ - stress functions of the ground state characterizing the untouched massif;

$\varphi^{00}(\zeta),\psi^{00}(\zeta),P^{00}(\zeta,\bar{\zeta})$ - stress functions of the additional stress state caused by the presence of a working;

$P^0(\zeta,\bar{\zeta})$- solution of the well-known Helmholtz equation of the form (2):

$$\Delta P+c^2P=0 \tag{2}$$

where Δ - Laplace operator, value c^2 - is the square of the modulus of the wave vector,

$\varphi^0(z),\psi^0(z)$ – Kolosov-Muskhelishvili stress functions for rock massif [8].

Let us write in the Cartesian coordinate system the basic (zero) equations of the planar moment theory of elasticity in the case of the absence of bulk ordinary and moment forces in an untouched massif:

- equilibrium equations according to the formula (3):

$$\frac{\partial\sigma_x^0}{\partial x}+\frac{\partial\tau_{yx}^0}{\partial y}=0;\ \frac{\partial\tau_{xy}^0}{\partial x}+\frac{\partial\sigma_y^0}{\partial y}=0;\ \frac{\partial\mu_x^0}{\partial x}+\frac{\partial\mu_y^0}{\partial y}+\tau_{xy}^0-\tau_{yx}^0=0 \tag{3}$$

- Hooke's law for a medium with couple stresses according to formula (4):

$$\varepsilon_x^0=\frac{1}{2G}\left[\sigma_x^0-\nu\left(\sigma_x^0+\sigma_y^0\right)\right], \qquad \gamma_{xy}^0=\frac{1}{4G}\left(\tau_{xy}^0+\tau_{yx}^0\right),\ \chi_x^0=\frac{\mu_x^0}{4G},\ \chi_y^0=\frac{\mu_y^0}{4G} \tag{4}$$

where the quantities $\sigma_x^0,\sigma_y^0,\tau_{xy}^0,\tau_{yx}^0\left(\tau_{xy}^0\neq\tau_{yx}^0\right)$ and μ_x^0,μ_y^0- components, respectively, of the main ordinary and couple stresses of the untouched massif;

$\varepsilon_x^0,\varepsilon_y^0,\gamma_{yx}^0$ – components of the main deformations from ordinary stresses;

χ_x^0,χ_y^0- components of the main deformations (curvature) from moment stresses in an intact massif;

E – Young's modulus, G – Poisson's ratio.

The deformations and the component of the rotation vector are related to the components of the displacement vector by formulas (5):

$$\varepsilon_x^0=\frac{\partial u^0}{\partial x},\ \varepsilon_x^0=\frac{\partial \upsilon^0}{\partial y},\ \gamma_{xy}^0=\frac{1}{2}\left(\frac{\partial\upsilon^0}{\partial x}+\frac{\partial u^0}{\partial y}\right),\ \chi_x^0=\frac{\partial\omega^0}{\partial x},\ \chi_y^0=\frac{\partial\omega^0}{\partial y},\ \omega^0=\frac{1}{2}\left(\frac{\partial\upsilon^0}{\partial x}-\frac{\partial u^0}{\partial y}\right) \tag{5}$$

III. Mathematical Model and Problem Solving Methodology

To solve the problem, analytical and numerical methods are used, as well as calculations using the Fortran programming language. The choice in favor of this particular programming language is dictated by the convenience and obtaining a more accurate solution of differential equations.

Let us construct some domains S and Σ on the plane, which are described by the corresponding analytic functions of the complex variables z and ζ. Let us assign to each point ζ of the region Σ a certain point z of the region S using relation (6):

$$z = \omega(\zeta) \tag{6}$$

where $\omega(\zeta)$ is a single-valued analytic function in some domain Σ on the plane of the complex variable $\zeta = \xi + i\eta$, *i* is the imaginary unit ($i^2 = -1$).

If the reverse correspondence is also possible, then there is a conformal mapping or transformation of the domain S onto Σ, and vice versa [7].

If the function (6) has a simple pole at some point, then the point $z = \infty$ corresponds to the point ζ, and the function $\omega(\zeta)$ will have the form $\omega(\zeta) = \frac{c}{\zeta} + f(\zeta)$ with a holomorphic function $f(\zeta)$ (c-const). The point ζ can become a potential bifurcation point or a critical point of the problem.

In addition, a holomorphic function can be expanded in a series with any required accuracy in the form (7):

$$f(\zeta) = a_0 + \frac{a_1}{\zeta} + \frac{a_2}{\zeta^2} + \frac{a_3}{\zeta^3} + \ldots \tag{7}$$

Such a transformation allows you to display any mathematical function from the Cartesian coordinate system to the polar one. The geometric interpretation of such an operation is to map the area of an arbitrary contour onto a circle of unit radius centered at the origin, which greatly simplifies calculations.

The advantage of the proposed method is obtaining numerical results in dimensionless units.

In an untouched rock mass, only vertical displacements take place, so the main stresses have the following form (8):

$$\sigma_x^0 = \lambda\sigma_y^0 = -\lambda\gamma H,\ \tau_{xy}^0 = \tau_{yx}^0 = 0,\ \mu_x^0 = \mu_y^0 = 0 \tag{8}$$

where $\lambda = \frac{\nu}{1-\nu}$ - side pressure coefficient, $\gamma -$ specific (volumetric) weight of a rock mass; H - the depth of the array point being considered. Now the main stress functions will be equal to (9):

$$\varphi^0(z) = \Gamma z = -\frac{\gamma \mathrm{H}(1+\lambda)}{4} z,\ \psi^0(z) = \Gamma' z = -\frac{\gamma \mathrm{H}(1-\lambda)}{2} z,\ \mathrm{P}^0(z,\bar{z}) = 0 \tag{9}$$

where $\Gamma, \Gamma' -$ stress distribution characteristics at infinity.

Let's construct an underground tunnel of an arbitrary transverse profile in a rock mass and find the components of ordinary and moment stresses, displacements and rotations in its vicinity. Such an array around a non-circular working is modeled under plane deformation conditions as an infinite isotropic elastic weightless plane with asymmetric stress tensors. It is weakened by a hole of some form in the plane of the complex variable z=x+iy and free from external forces and moments.

The calculation scheme of the problem is shown in Figure 1, where a non-circular hole of arbitrary shape is located in the plane of the complex variable z=x+iy.

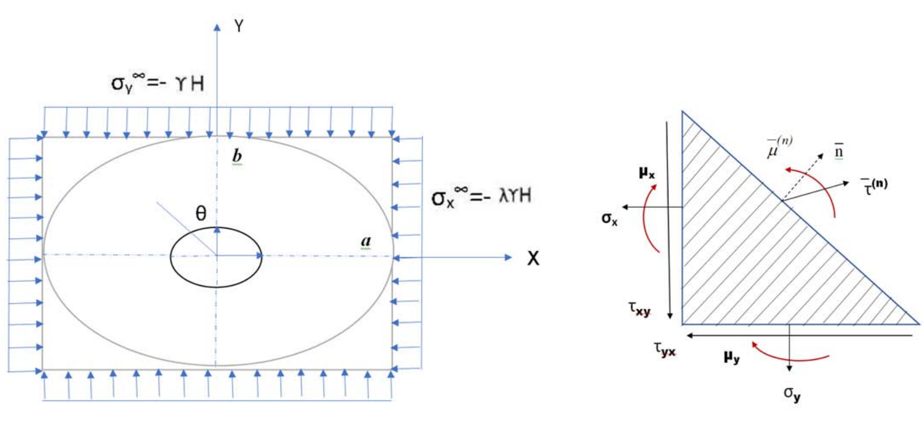


Fig. 1: Calculation Scheme of the Problem

To solve the problem, we pass to the region of the complex variable Σ and conformally map the entire infinite region outside the hole onto the exterior of the unit circle in the variable $\zeta = \xi + i\eta = \rho e^{i\theta}$ plane using the mapping function (10):

$$z = \omega(\zeta) = R[\zeta + \varepsilon\phi(\zeta)] = R(\zeta + \varepsilon\zeta^{-1}) \qquad (10)$$

where ρ, θ are polar coordinates.
Condition (11) is required:

$$1 + \varepsilon\phi'(\zeta) \neq 0 \text{ при } |\zeta| \geq 1 \qquad (11)$$

Here $\phi(\zeta) = \sum_{1}^{N}(\alpha_n + i\beta_n)\zeta^{-n}$ $(\alpha_n, \beta_n, R - const)$, $R = \frac{a+b}{2}, \varepsilon = \frac{a-b}{a+b}$ (a,b – semiaxes of an ellipse),

ε - small numerical parameter, is in the interval $-1 \leq \varepsilon \leq 1$ and characterizes the deviation of a given hole from a circular.

To solve the problem, initial and boundary conditions are set.

1. Initial conditions at t=0 are given by formula (12):

$$\upsilon_\rho = 0, \upsilon_\theta = 0, \omega_{\rho\theta} = 0 \qquad (12)$$

2. Boundary conditions on the contour of a unit circle at are $\rho = 1$ given by formula (13):

$$\sigma_\rho - i\tau_{\rho\theta} = 0\,,\ \mu_\rho = 0 \qquad (13)$$

where $\upsilon_\rho, \upsilon_\theta, \omega_{\rho\theta}$ - displacement and rotation components, $\sigma_\rho, \tau_{\rho\theta}, \mu_\rho$ - stress components on the circuit.

The functions of the main stresses in the displayed area according to formula (8) will take the following form (14):

$$\varphi^\circ(\zeta) = \Gamma\omega(\zeta), \psi^\circ(\zeta) = \Gamma'\omega(\zeta), \mathrm{P}^\circ(\zeta, \bar{\zeta}) = 0 \qquad (14)$$

The additional stress functions are found from the boundary conditions and, according to the Laurent theorem, can be represented in the region outside the hole by uniformly convergent power series (15):

$$\varphi^{00}(\zeta) = a_n\zeta^{-n}, \psi^{00}(\zeta) = \sum_{0}^{\infty} b_n\zeta^{-n},\ \mathrm{P}^{00}(\zeta, \bar{\zeta}) = \sum_{-\infty}^{\infty} p_n \mathrm{K}_n(cR\rho)e^{-in\theta} \qquad (15)$$

where $\mathrm{K}_n(cR\rho)$– modified Bessel function of the second kind of the nth order of the imaginary argument (McDonald function) [10]. To find the derivatives of the function, the following well-known recursive formulas (16) were used:

$$\begin{aligned} &\frac{\partial^k}{\partial\zeta^k}\left[\mathrm{K}_n(cR\rho)e^{\pm in\theta}\right] = \left(-\frac{cR}{2}\right)^k K_{n\mp k}e^{\pm i(n\mp k)\theta}, \\ &\frac{\partial^k}{\partial\bar{\zeta}^k}\left[\mathrm{K}_n(cR\rho)e^{\pm in\theta}\right] = \left(-\frac{cR}{2}\right)^k \mathrm{K}_{n\pm k}e^{\pm i(n\pm k)\theta} \end{aligned} \qquad (16)$$

After carrying out the required transformations, the stress functions $\varphi(\zeta), \psi(\zeta)$ are obtained, which will be holomorphic from ζ outside the circle of unit radius, and the function $\mathrm{P}(\zeta, \bar{\zeta})$ will satisfy equation (17):

$$\Delta\mathrm{P} - c^2|\omega'(\zeta)|^2\mathrm{P} = 0,$$

$$\Delta = \frac{\partial^2}{\partial\zeta^2} + \frac{\partial^2}{\partial\eta^2} = 4\frac{\partial^2}{\partial\zeta\partial\bar{\zeta}} \qquad (17)$$

Potentials $\varphi(\zeta), \psi(\zeta), \mathrm{P}(\zeta, \bar{\zeta})$ outside the contour of the hole will be sought in the form of series in powers of the small parameter ε according to formula (18) and we will restrict ourselves to the zeroth and first approximations:

$$\varphi(\zeta) = \sum_{0}^{\infty}\varepsilon^n\varphi_n(\zeta), \quad \psi(\zeta) = \sum_{0}^{\infty}\varepsilon^n\psi_n(\zeta), \quad \mathrm{P}(\zeta, \bar{\zeta}) = \sum_{0}^{\infty}\varepsilon^n\mathrm{P}_n(\zeta, \bar{\zeta}) \qquad (18)$$

We expand the functions included in the boundary conditions (13) into functional series in powers of the small parameter ε according to (18) and compare the expressions with each other for its equal powers. For expansion, we also use the following well-known power series (19):

$$\frac{1}{(1+x)} = \sum_{0}^{\infty}(-1)^k x^k = 1 - x + x^2 - x^3 + \ldots \qquad (19)$$

Differentiating it, we get another power series we need (20):

$$\frac{1}{(1+x)^2}=\sum_{0}^{\infty}(-1)^{k+1}kx^{k-1}=1-2x+3x^2-\dots \qquad (20)$$

Also, equating the expressions with equal powers of the small parameter $\sigma^n=e^{in\theta}$, we obtain the following system for determining the solution of the Helmholtz equation - functions Pn (21):

$$\left.\begin{aligned}&\Delta P_0-c^2R^2P_0=0\\&\Delta P_1-c^2R^2P_1=c^2R^2(\phi'+\bar{\phi}')P_0\\&\Delta P_n-c^2R^2P_n=c^2R^2\left[(\phi'+\bar{\phi}')P_{n-1}+|\phi'|^2P_{n-2}\right],n\geq 2\end{aligned}\right\} \qquad (21)$$

In this case, we restrict ourselves to the zero and first approximations (22)-(24):

$$\begin{aligned}\varphi(\zeta)&=\varphi_0(\zeta)+\varepsilon\varphi_1(\zeta)=\Gamma\omega(\zeta)+\sum_{1}^{\infty}a_n\zeta^{-n}=\\&=\Gamma(\zeta+\varepsilon\zeta^{-1})+[a_1^{(0)}+\varepsilon a_1^{(1)}]\zeta^{-1}+[a_3^{(0)}+\varepsilon a_3^{(1)}]\zeta^{-3}\\&=\Gamma\zeta+\frac{a_1^{(0)}}{\zeta}+\varepsilon[\frac{\Gamma}{\zeta}+\frac{a_1^{(1)}}{\zeta}+\frac{a_3^{(1)}}{\zeta^3}]\end{aligned} \qquad (22)$$

$$\psi(\zeta)=\Gamma'\zeta+\frac{b_1^{(0)}}{\zeta}+\frac{b_3^{(0)}}{\zeta^3}+\varepsilon\left[\frac{\Gamma'}{\zeta}+\frac{b_1^{(1)}}{\zeta}+\frac{b_3^{(1)}}{\zeta^3}+\frac{b_5^{(1)}}{\zeta^5}\right] \qquad (23)$$

$$\begin{aligned}P(\zeta,\bar{\zeta})&=\left[p_2^{(0)}e^{2i\theta}+\bar{p}_2^{(0)}e^{-2i\theta}\right]K_2(cR\rho)+\\&+\varepsilon\left\{\Gamma'\frac{F}{1+F}\frac{c^2R^2}{6cK_1(cR)}K_2(cR\rho)\sin 4\theta+\right.\\&+\varepsilon\left\{\left[p_2^{(1)}e^{2i\theta}+\bar{p}_2^{(1)}e^{-2i\theta}\right]K_2(cR\rho)\right.\end{aligned} \qquad (24)$$

where $a_n^{(0)},b_n^{(0)},p_n^{(0)}(n\geq 1)$ - zero approximation coefficients;

$a_n^{(1)},b_n^{(1)},p_n^{(1)}(n\geq 1)$ - coefficients for the first approximation.

As a result of expansion into series (18) and equating the coefficients for the same powers of a small parameter, we obtain a sequence of boundary value problems for a circular hole $(\omega=R\xi)$:

$$\left.\begin{aligned}&\varphi_0+\sigma\overline{\varphi_0'}+\bar{\psi}+\frac{m}{R}\overline{\Phi_0'}-\frac{2i}{R}\frac{\partial P_0}{\partial\bar{\sigma}}=0\\&\mathrm{Re}\left\{\frac{i}{\sigma}\left[m\overline{\Phi_0'}-2i\frac{\partial P_0}{\partial\bar{\sigma}}\right]\right\}=0\end{aligned}\right\} \qquad (25)$$

$$\left.\begin{aligned}&\varphi_n+\sigma\overline{\varphi_n'}+\overline{\psi_n}+\frac{m}{R}\overline{\Phi_n'}-\frac{2i}{R}\frac{\partial P_n}{\partial\bar{\sigma}}+\varphi_{n-1}\overline{\phi'}+\overline{\varphi_{n-1}'}\phi+\overline{\psi_{n-1}\phi'}=0\\&\mathrm{Re}\left\{\frac{i}{\sigma}\left[m\overline{\Phi_n'}-2i\frac{\partial P_n}{\partial\bar{\sigma}}\right]\right\}=0 \qquad (n=1,2,3...)\end{aligned}\right\} \qquad (26)$$

IV. Results and Performance Analisis

A feature of problem (24) is that, as a result of the solution, we obtain only the imaginary parts of the complex variable $\zeta=\xi+i\eta$.

1. The stress functions of the state of the array in the zeroth approximation.

In the displayed area Σ on the contour of the working at ρ=1, the components of stresses and displacements are calculated as follows:

- for the main stress state according to the formulas (27):

$$\begin{aligned}&\sigma_\rho^{(0)0}=-\frac{\gamma H}{2}\left[(1+\lambda)-(1-\lambda)\cos 2\theta\right],\\&\sigma_\theta^{(0)0}=-\frac{\gamma H}{2}\left[(1+\lambda)+(1-\lambda)\cos 2\theta\right],\\&\tau_{\rho\theta}^{(0)0}=-\frac{\gamma H}{2}(1-\lambda)\sin 2\theta\\&\tau_{\theta\rho}^{(0)0}=-\frac{\gamma H}{2}(1-\lambda)\sin 2\theta,\\&\upsilon_\rho^{(0)0}=-\frac{\gamma H\rho}{4G}\left[(1-2\nu)(1+\lambda)-(1-\lambda)\cos 2\theta\right],\\&\upsilon_\theta^{(0)0}=-\frac{\gamma H\rho}{4G}(1-\lambda)\sin 2\theta,\\&\mu_\rho^0=\mu_\theta^0=\omega_{\rho\theta}^0=0,\end{aligned} \qquad (27)$$

- for an additional stress state according to formulas (28):

$$\begin{aligned}&\sigma_p^{(0)00}=\frac{\gamma H}{2}\{(1+\lambda)-(1-\lambda\cos 2\theta)\},\\&\sigma_\theta^{(0)00}=-\frac{\gamma H}{2}\left\{(1+\lambda)+\left[3-\frac{4F}{1+F}\right](1-\lambda)\cos 2\theta\right\},\\&\tau_{p\theta}^{(0)00}=\frac{\gamma H}{2}(1-\lambda)\sin 2\theta\\&\tau_{\theta_p}^{(0)00}=\frac{\gamma H}{2}\left\{1-\frac{2F}{1+F}\left(2+cR\frac{K_0(cR)}{K_1(cR)}\right)\right\}(1-\lambda)\sin 2\theta,\\&\mu_\rho^{(0)00}=0,\mu_\theta^{(0)00}=\gamma HR\frac{F}{1+F}(1-\lambda)\cos 2\theta\\&\upsilon_\rho^{(0)00}=-\frac{\gamma HR}{4G}\left\{(1+\lambda)-\left[3-4\nu-\frac{4(1-\nu)F}{1+F}\right](1-\lambda)\cos 2\theta\right\}\\&\upsilon_\theta^{(0)00}=-\frac{\gamma HR}{4G}\left\{3-4\nu-\frac{4(1-\nu)F}{1+F}\right\}(1-\lambda)\sin 2\theta\\&\omega_{\rho\theta}^{(0)00}=\frac{\gamma H}{8G}c^2R^2\frac{F}{1+F}(1-\lambda)\sin 2\theta\end{aligned} \qquad (28)$$

where
$$F=\frac{8(1-\nu)}{4+c^2R^2+2cR[\mathrm{K}_0(cR)/\mathrm{K}_1(cR)]} \tag{29}$$

2. The stress functions of the state of the array in the first approximation.

- the components of the main stresses on the contour of the working at ρ=1 will be found by the formulas (30):

$$\begin{aligned}
&\sigma_{\rho}^{(1)0}=\frac{\gamma H(1-\lambda)}{2}\{\cos 4\theta-1\},\\
&\sigma_{\theta}^{(1)0}=-\frac{\gamma H(1-\lambda)}{2}\{\cos 4\theta-1\},\\
&\tau_{\rho\theta}^{(1)0}=-\frac{\gamma H(1-\lambda)}{2}\sin 4\theta,\\
&\tau_{\theta\rho}^{(1)00}=-\frac{\gamma H(1-\lambda)}{2}\sin 4\theta,\\
&\upsilon_{\rho}^{(1)0}=-\frac{\gamma HR}{4G}\left\{-\frac{1-\lambda}{2}+(1+\lambda)(1-2\cos 2\theta)-\frac{1-\lambda}{2}\cos 4\theta\right\},\\
&\upsilon_{\theta}^{(1)0}=-\frac{\gamma HR}{4G}\left\{2(1+\lambda)(2\nu-1)\sin 2\theta+\frac{1-\lambda}{2}\sin 4\theta\right\}\\
&\mu_{\rho}^{(1)0}=\mu_{\theta}^{(1)0}=\omega_{\rho\theta}^{(1)}=0
\end{aligned} \tag{30}$$

- the components of additional stresses on the working contour at $\rho=1$ can be found by the formulas (31):

$$\begin{aligned}
&\sigma_{\rho}^{(1)00}=-\frac{\gamma H(1-\lambda)}{2}[\cos 2\theta-1]\\
&\sigma_{\theta}^{(1)00}=-\frac{\gamma H(1-\lambda)}{2}-\frac{2\gamma H(1+\lambda)}{1+F}\cos 2\theta-\\
&-\frac{2\gamma H(1-\lambda)}{1+F}\left[\frac{3-F}{4}-\frac{4(1-\nu)R_2}{R_1}\right]\cos 4\theta
\end{aligned} \tag{31}$$

$$\tau_{\rho\theta}^{(1)00}=\frac{\gamma H(1-\lambda)}{2}\sin 4\theta \quad \tau_{\theta\rho}^{(1)00}=\tau_{\rho\theta}^{(1)00}+c^2R_1$$

$$\mu_{\rho}^{(1)00}=0$$

$$\mu_{\theta}^{(1)00}=\frac{\gamma HR}{2}\frac{F}{1+F}\{1-\lambda+2(1+\lambda)\cos 2\theta+$$

$$+(1-\lambda)\left[1+\frac{c^2R^2}{6}\frac{K_1+K_3}{K_1}\frac{R_2}{R_1}\right]\cos 4\theta\}$$

at $R_1=1+\dfrac{96(1-\nu)K_3}{c^2R^2[K_3+K_5]}$,

$$R_2=\frac{K_1-\frac{2}{cR}K_2}{K_1+K_3}+\frac{\left(\frac{24}{c^2R^2}-1\right)K_3}{K_3+K_5}, \tag{32}$$

From the formulas (27)-(32) obtained above for the main and additional stress state, it is clearly seen that the effect of moment stresses affects only the additional stress state of the mass, which consists of the classical elastic part and the part due to the influence of the new elastic constant λ included in the quantity $\dfrac{F}{1+F}$.

According to formulas (27) - (32), we will carry out numerical calculations using Fortran, we will build graphs with initial data that characterize the physical properties of the siltstone rock [11]:

$E=0{,}62\cdot 10^{10}\,МПа$, ν=0.20, α=0.726, $\delta=0.0094\,сек^{\alpha-1}$, λ=0.25, cR=3, Macdonald functions of the second kind $K_0=0.0347$, $K_1=0.0402$, $K_2=0.0615$, $K_3=0.1222$, $K_4=0.3059$, $K_5=0.9378$, $K6=3.4318$. The polar angle is taken in the interval $0\le\theta\le 2\pi$.

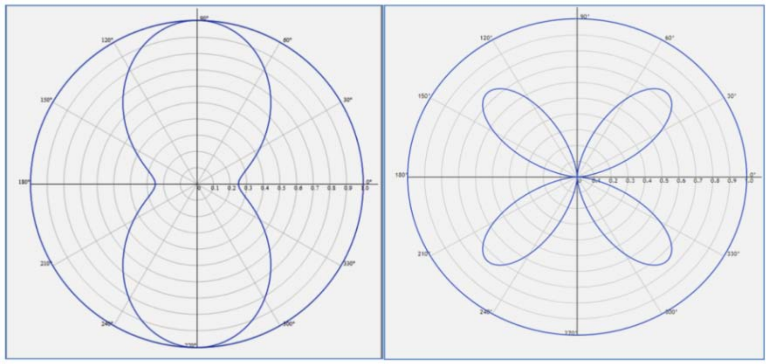
Table 1 shows the dimensionless values of the main stresses of a solid rock mass in the zero and first approximation, calculated using formulas (25) and (26).

Table 1: Basic Stresses of a Solid Rock Mass in the Zero and First Approximation

θ, degree	$\frac{\sigma_{\rho}^{(0)0}}{\gamma H}$	$\frac{\sigma_{\rho}^{(1)0}}{\gamma H}$	$\frac{\sigma_{\theta}^{(0)0}}{\gamma H}$	$\frac{\sigma_{\theta}^{(1)0}}{\gamma H}$	$\frac{\tau_{\rho\theta}^{(0)0}}{\gamma H}$	$\frac{\tau_{\rho\theta}^{(1)0}}{\gamma H}$	$\frac{\tau_{\theta\rho}^{(0)0}}{\gamma H}$	$\frac{\tau_{\theta\rho}^{(1)0}}{\gamma H}$
0	0.250	0.000	1.000	0.000	0.000	0.000	0.000	0.000
15	0.300	0.188	0.949	-0.188	0.188	0.325	0.188	0.325
30	0.438	0.563	0.813	-0.563	0.325	0.325	0.325	0.325
45	0.625	0.750	0.625	-0.750	0.375	0.000	0.375	0.000
60	0.813	0.563	0.438	-0.563	0.325	-0.325	0.325	-0.325
75	0.949	0.188	0.302	-0.188	0.188	-0.325	0.188	-0.325
90	1.000	0.000	0.250	0.000	0.000	0.000	0.000	0.000
105	0.949	0.188	0.302	-0.188	-0.188	0.325	-0.188	0.325
120	0.812	0.563	0.438	-0.563	-0.325	0.325	-0.325	0.325
135	0.625	0.750	0.625	-0.750	-0.375	0.000	-0.375	0.000
150	0.438	0.563	0.813	-0.563	-0.325	-0.325	-0.325	-0.325
165	0.300	0.188	0.949	-0.188	-0.188	-0.325	-0.188	-0.325
180	0.250	0.000	1.000	0.000	0.000	0.000	0.000	0.000

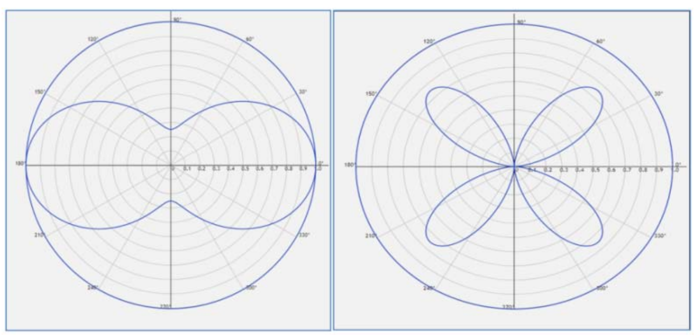
Figure 2 shows the main radial stresses in the zero and first approximation of a solid massif, which are distributed symmetrically about the coordinate axes, and in this case the rock mass experiences only compression. The value of the radial stress at the upper points of the contour is 4 times greater than the stresses at the lateral points, and the circumferential stresses, on the contrary, are greater.

Figure 3 shows the main circumferential stresses in the zero and first approximation.



a) $-\frac{\sigma_{\rho}^{(0)o}}{\gamma H}$ b) $-\frac{\sigma_{\rho}^{(1)o}}{\gamma H}$

Fig. 2: Basic Radial Stresses in Zero and First Approximation

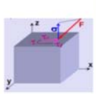


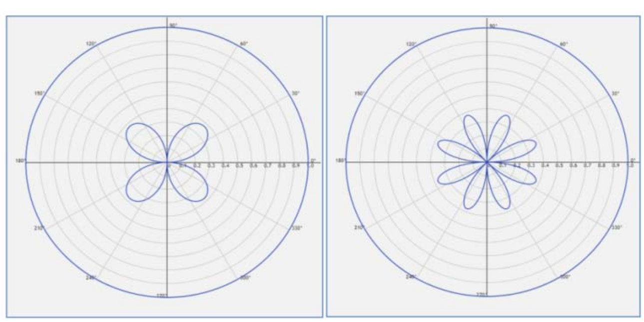
a) $-\frac{\sigma_{\theta}^{(0)o}}{\gamma H}$ b) $-\frac{\sigma_{\theta}^{(1)o}}{\gamma H}$

Fig. 3: Basic Circumferential Stresses in Zero and First Approximation

From Figure 3, we can conclude that the stresses are symmetrical with respect to the coordinate axes and the rock mass experiences only compression.

Figure 4 shows that the main shear stresses in the zero and first approximation are symmetrical about the axis of the bisector of the first quarter. In this case, in the first and third quarters, the array experiences only compression, and in the rest of the area, tension.





a) $-\frac{\tau_{\rho\theta}^{(0)o}}{\gamma H}$ b) $-\frac{\tau_{\rho\theta}^{(1)o}}{\gamma H}$

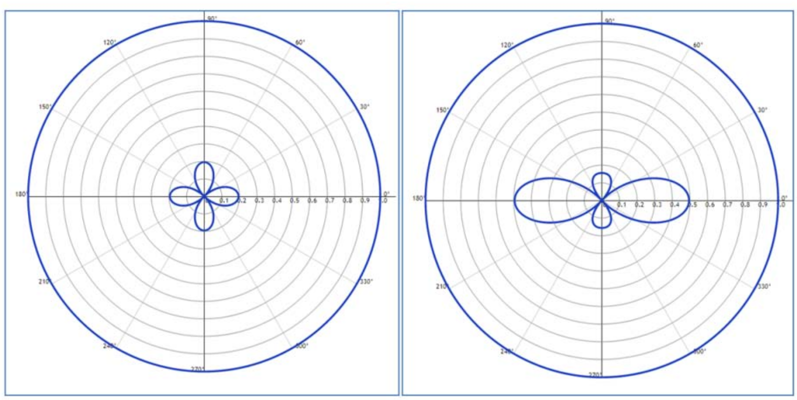
Fig. 4: Basic Shear Stresses in Zero and First Approximation

Table 2 shows the values of additional stresses at the initial moment of time, calculated by formulas (22) and (24) in dimensionless units.

Figure 5 shows additional moment hoop stresses in zero and first approximation.

Table 2: Additional Stresses in the Rock Mass Around the Loose Elliptical Working

θ, degree	$-\frac{\sigma_\rho^{(0)00}}{\gamma H}$	$-\frac{\sigma_\rho^{(1)00}}{\gamma H}$	$-\frac{\sigma_\theta^{(0)00}}{\gamma H}$	$-\frac{\sigma_\theta^{(1)00}}{\gamma H}$	$-\frac{\tau_{\rho\theta}^{(0)00}}{\gamma H}$	$-\frac{\tau_{\rho\theta}^{(1)00}}{\gamma H}$	$-\frac{\tau_{\theta\rho}^{(0)00}}{\gamma H}$	$-\frac{\tau_{\theta\rho}^{(1)00}}{\gamma H}$	$\frac{\mu_\theta^{(0)00}}{\gamma HR}$	$\frac{\mu_\theta^{(1)00}}{\gamma HR}$
0	-0.250	0.000	1.359	2.624	0.000	0.000	0.000	0.000	-0,195	0,519
15	-0.302	-0.188	1.261	2.177	-0.1885	-0.325	0.261	-0.543	-0,169	0,329
30	-0.434	-0.563	0.992	1.099	-0.3258	-0.325	0.452	0.005	-0,098	0,462
45	-0.625	-0.750	0.625	-0.025	-0.375	0.000	0.522	1.495	0,000	0,216
60	-0.813	-0.563	0.258	-0.749	-0.325	0.325	0.452	2.585	0,098	0,152
75	-0.949	-0.188	-0.011	-1.027	-0.188	0.325	0.261	2.038	0,169	0,001
90	-1.000	0.000	-0.109	-1.074	0.000	0.000	0.000	0.000	0,195	-0,065
105	-0.949	-0.188	-0.011	-1.027	0.188	-0.325	-0.261	-2.038	0,169	0,057
120	-0.813	-0.563	0.258	-0.749	0.325	-0.325	-0.452	-2.585	0,098	0,045
135	-0.625	-0.750	0.625	-0.025	0.375	0.000	-0.522	-1.495	0,000	0,364
150	-0.437	-0.563	0.992	1.099	0.325	0.325	-0.452	-0.005	-0,098	0,287
165	-0.301	-0.188	1.261	2.177	0.188	0.325	-0.261	0.5437	-0,169	0,513
180	-0.250	0.000	1.359	2.624	0.000	0.000	0.000	0.000	-0,195	0,342



$\frac{\mu_\theta^{(0)00}}{\gamma HR}$ $\frac{\mu_\theta^{(1)00}}{\gamma HR}$

Fig. 5: Basic Additional Moment Stresses in Zero and First Approximation

Analysis of Figure 5 shows that the first distribution is symmetrical, while the second distribution is symmetrical about the X and Y axes and asymmetric about the axes of the first and third, as well as the second and fourth quarters.

Total stresses, displacements and rotations for an elliptical working are calculated as the sum of the zero and first approximations (main and additional).

V. Conclusion

Thus, the task of determining the potential stress functions that affect a certain system is completely solved. The advantage of the study is that the task is complicated by taking into account the moment stresses that cause deformations of curvature and rotation in the system. This increases the risk of loss of stability of the system and the rate of its destruction.

The use of such methods as mathematical and computer modeling, the use of numerical methods, contributed to obtaining adequate solutions to the problem.

The graphical implementation of the obtained numerical results makes it possible to see the critical zones in which the system experiences the greatest pressure from external forces. As a result, the system can lose its stability and go into a nonequilibrium state. It is in these areas that engineering construction requires urgent measures such as strengthening and strengthening mechanisms with effective technological solutions to avoid possible catastrophic collapses.

The adoption of preventive measures to identify the risks of disasters by identifying critical areas and tools for their neutralization can be widely used in the analysis of the behavior of complex systems that are affected by some external forces.

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