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Otimal Control of Time-Delay Systems

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I. Introduction

It is clear for control engineers that handling time delay requires special attention from the early days of the control history. The time delay is an uncancelable, invariant property of the process. The early goals tried to find design procedures which allow the selection of the regulator quasi independently from the delay. An early success story was the SMITH predictor or regulator [1].

Consider a continuous time delay process given by its transfer function

$$P(s) = P_+(s)\bar{P}_-(s) = P_+(s)e^{-sT_d}\ ;\ P = P_+\bar{P}_- = P_+e^{-sT_d} \quad (1)$$

where T_d is the time delay, P_+ is stable and $P_- = e^{-sT_d}$ is the *Inverse-Unstable-Unrealizable* (*IUU*) part of the process, respectively. The original SMITH predictor is shown in Fig. 1, where r is the reference signal and y is the process output.

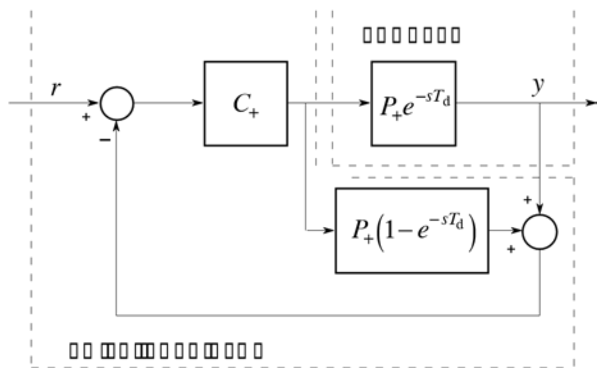

Fig.1: The Block-Scheme of the SMITHPredictor

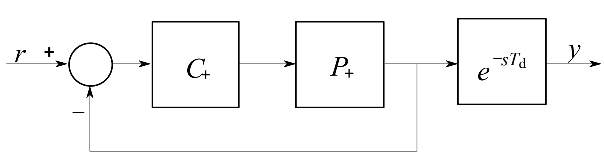

Fig. 2: Equivalent Block-Scheme of the SMITHPredictor

It is easy to check that the SMITH predictor is equivalent to the scheme shown in Fig. 2. This figure clearly shows that the regulator C_+ can be designed to the delay free P_+, independently of the time delay T_d. This scheme explains why the SMITH predictor is also called SMITH regulator [8], [9], [10]. The whole procedure is, of course, not independent of T_d, because the predictor scheme contains block depending on the delay.

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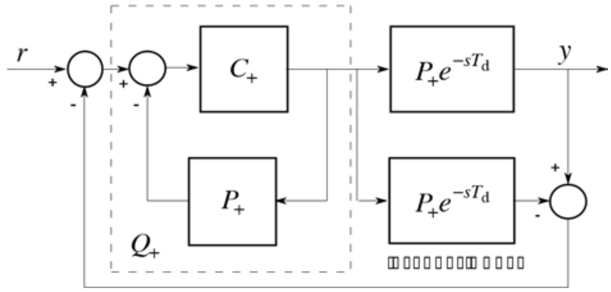

Fig. 3: *IMC* form of the SMITHPredictor

It is possible to redraw the SMITH predictor into further schemes, which allow special interpretations. Fig. 3. shows another equivalent scheme what corresponds to the well known*Internal Model Control* (*IMC*) scheme and principle. Fig. 4. presents the resulting closed-loop with the serial regulator C_s equivalent to the application of the SMITH predictor.

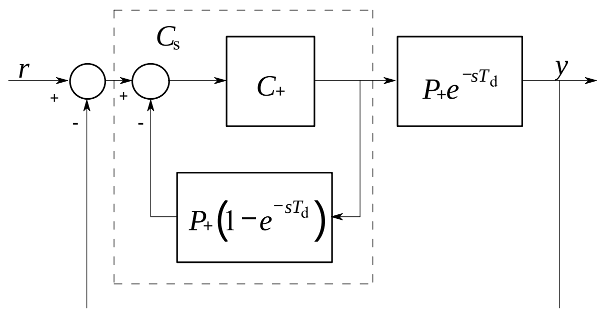

Fig. 4: The Resulting Closed-Loop of the SMITHPredictor

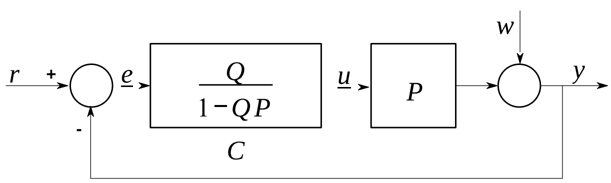

Fig. 5: YOULA-Parameterized Closed-Loop

II. The YoulaParameterization

A YOULA-parameterized (*YP*) closed-loop [4], [8] is shown in Fig. 5, where *e* is the error, *u* is the regulator output and *w*is the output disturbance signal, respectively.

Here the plant *P* is stable and the *All-RealizableStabilizing* (*ARS*) regulator is

$$C = \frac{Q}{1-QP} \tag{2}$$

The closed-loop transfer function or *Complementary Sensitivity Function* (*CSF*)

$$T = \frac{CP}{1+CP} = QP \tag{3}$$

which is linear in the stable YOULA parameter *Q*.

It is well known that the *YP* regulator corresponds to the classical *IMC* structure shown in Fig. 6, where *r* is the reference signal, *u* is the regulator output, *y* is the output signal and *w* is the output disturbance signal, respectively. If there is no disturbance and the internal model is equal to the process transfer function, the signal fed back to the reference signal is zero, and the forward path *QP* determines the reference signal tracking. The feedback loop rejects the effect of the disturbance and of the plant/model mismatch.

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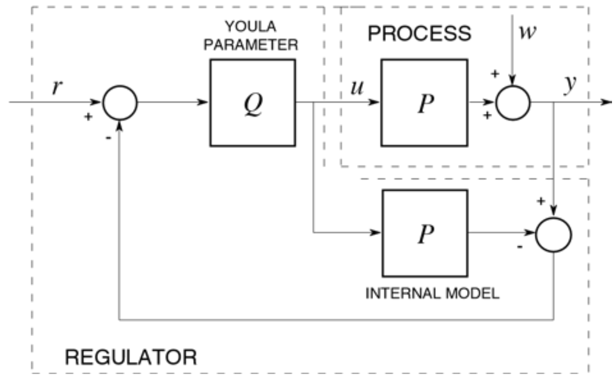

Fig. 6: *IMC* form of the *YP* Closed- Loop

It can also be well seen that Q_+ in Fig. 3 corresponds to the YOULA parameter. For a more detailed comparison consider the extension of *YP* regulator for more general case next.

III. A *G2DOF* Controller for Stable LinearPlants

The first systematic method introducing the *generic twodegree of freedom* (*G2DOF*) scheme was presented in [5], [8], [9], [10] when the process is open-loop stable and it is allowed to cancel the stable process poles, which case occurs at many practical tasks. *2DOF* in this approach means that the dynamics of reference signal tracking and that of disturbance rejecting are different. This framework and topology is based on the *YP* providing *ARS* regulators for open-loop stable plants and capable to handle the plant time-delay, too.

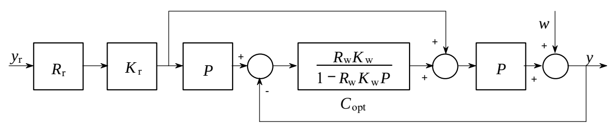

Fig. 7: The *generic2DOF* (*G2DOF*) Control System

A *G2DOF* control system is shown in Fig. 7 for the stable process

$$P = P_+ \overline{P}_- = P_+ P_- e^{-sT_d} \qquad (4)$$

which is more general than what was used in (1), because here P_+ is stable and <u>I</u>*nverse*-<u>S</u>*table*-<u>R</u>*ealizable* (*ISR*), P_- is <u>I</u>*nverse*-<u>U</u>*nstable*-<u>U</u>*nrealizable* (*IUU*).

The optimal *ARS* regulator of the *G2DOF* scheme can be given by an explicit form

$$C_{opt} = \frac{R_w K_w}{1 - R_w K_w P} = \frac{Q_o}{1 - Q_o P} = \frac{R_w G_w P_+^{-1}}{1 - R_w G_w P_- e^{-sT_d}} \qquad (5)$$

where

$$Q_o = Q_w = R_w K_w = R_w G_w P_+^{-1} \qquad (6)$$

is the associated optimal *Y-parameter.* Furthermore

$$Q_r = R_r K_r = R_r G_r P_+^{-1} \;;\; K_w = G_w P_+^{-1} \;;\; K_r = G_r P_+^{-1} \qquad (7)$$

The *YP* regulator (5) can be considered the generalization of the TRUXAL-GUILLEMIN [2], [8], [9], [10] method for stable processes.

It is interesting to see how the transfer characteristics of the closed-loop look like:

$$y = R_r K_r P y_r + (1 - R_w K_w P) w = R_r G_r P_- e^{-sT_d} y_r + + \left(1 - R_w G_w P_- e^{-sT_d}\right) w = y_t + y_d \qquad (8)$$

where y_t is the tracking (servo) and y_d is the regulating (or disturbance rejection) independent behaviors of the closed- loop response, respectively. So the delay e^{-sT_d} and P_-cannot be eliminated, consequently the ideal design goals R_r and R_w are biased by $G_r P_-$ and $G_w P_-$. Here R_r and R_w are assumed stable and usually strictly proper transfer functions, that are partly capable to place desired poles in the tracking and the regulatory transfer functions, furthermore they are usually referred as reference signal and output disturbance predictors. They can even be called as reference models, so reasonably $R_r(\omega=0) = 1$ and $R_w(\omega=0) = 1$ are selected. The unity gain of $R_w$ensures integral action in the regulator, which is maintained if the applied optimization provides $G_w P_-(\omega=0) = 1$.

The role of R_r and R_w (predictors or filters) is threefold.

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They prescribe the tracking and regulatory properties of the control loop. They influence the magnitude of the actuating signal and also influence the robustness properties of the control system.

An interesting result was found [6] that the optimization of the *G2DOF* scheme can be performed in $H_2$and H_∞ norm spaces by the proper selection of the serial embedded filters G_r and G_w attenuating the influence of the invariant process factor P_-. Using H_2norm a *Diophantine-equation* (*DE*) should be solved to optimize these filters. If the optimality requires a H_∞ norm, then the NEVANLINNA-PICK (*NP*) approximation is applied.

After some straightforward block manipulations the *G2DOF* control system can be transformed to another form shown in Fig. 8, which is the generalized version of the classical *IMC* scheme in Fig. 6.

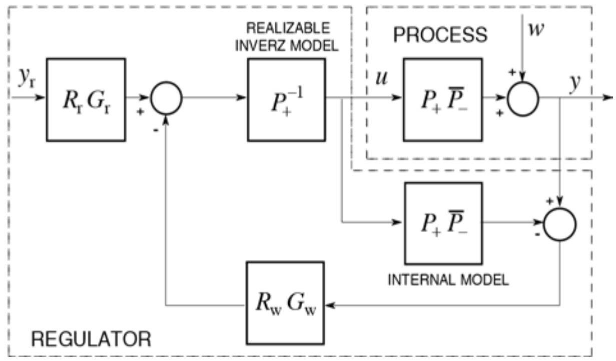

Fig. 8: The generalized *IMC* form of the *G2DOF* control system

IV. Smith Predictor as a Subclass of *G2DOF* Controllers

The previous two sections clearly show that the SMITH predictor is a special subclass of the *G2DOF* controllers with a *YP* parameterized regulator

$$Q_+ = \frac{C_+}{1+C_+P_+} = \frac{C_+P_+}{1+C_+P_+}P_+^{-1} = \frac{L_+}{1+L_+}P_+^{-1} = R_+P_+^{-1} \qquad (9)$$

if C_+ is stabilizing P_+, i.e., the delay free part of the process. Here the special *CSF*

$$T_+ = R_+ = \frac{L_+}{1+L_+} \qquad (10)$$

characterizing the closed-loop in Fig. 2 is the reference model R_+ and $L_+ = C_+P_+$ is its loop transfer function.

It is also easy to see that the resulting serial regulator of the SMITH predictor in Fig. 4 is

$$C_s = \frac{Q_+}{1-Q_+P_+e^{-sT_d}} = \frac{C_+}{1+C_+P_+\left(1-e^{-sT_d}\right)} = C_+K_S \qquad (11)$$

This formula presents the possible way of realization for a continuous-time (CT) case. Here K_S denotes a serial factor modifying the original C_+ regulator of the SMITH predictor

$$K_S = \frac{1}{1+C_+P_+\left(1-e^{-sT_d}\right)} = \frac{1}{1+L_+\left(1-e^{-sT_d}\right)} \qquad (12)$$

At the stability limit cross over frequency ω_c, where $L_+ = -1$ the factor K_S takes a considerable positive phase advance into the closed-loop

$$K_S = \frac{1}{1+(-1)\left(1-e^{-sT_d}\right)} = \frac{1}{1-1+e^{-sT_d}} = e^{sT_d}\Big|_{\omega_c} = e^{j\omega_c T_d} \qquad (13)$$

This is the simple physical explanation of the success of the SMITH predictor [3].

Some early evaluations state that unfortunately the SMITH predictor is only good for tracking and not for disturbance rejection. This evaluation is wrong. The SMITH regulator was proposed for a one-degree of freedom (*1DOF*) closed-loop, so it is naturally not for *2DOF* purposes. The real problem of the SMITH regulator is that it allows the design of the closed-loop only via an indirect way by selecting $R_+ = T_+$, while the design procedure of the *G2DOF* scheme gives a direct procedure to design the independent tracking and disturbance rejection properties. This means that the original idea of SMITH was that a classical design of T_+ is necessary for the proper application. One must know that the YOULA parameterization and its application for regulator design was unknown for Otto SMITH when he invented his predictor.

V. The Discrete-Time Version of *G2DOF* Controllers

Although (11) suggests a proper way how to realize the SMITH regulator, it is not realistic to build any regulator containing the e^{-sT_d} delay element for continuous-time case. In the practice only the discrete-time (DT) version can be applied by computer realization. Consider the DT model of the CT process in the form of its pulse transfer function given by

$$G\left(z^{-1}\right)=G_+\left(z^{-1}\right)\overline{G}_-\left(z^{-1}\right)=G_+\left(z^{-1}\right)G_-\left(z^{-1}\right)z^{-d} \quad G=G_+\overline{G}_-=G_+G_-z^{-d} \tag{14}$$

where G_+ is stable and *ISR*, G_- is *IUU* and z^{-d} corresponds to the discrete time-delay, where *d* is the integer multiple of the sampling time. (In a practical case the factor G_- can incorporate the underdamped zeros and the neglected poles providing realizability, too). The optimal *ARS* regulator of the *G2DOF* scheme can be given now by

$$C_o=\frac{R_wK_w}{1-R_wK_wS}=\frac{Q_o}{1-Q_oG}=\frac{R_wG_wG_+^{-1}}{1-R_wG_wG_-z^{-d}} \tag{15}$$

which corresponds to the CT case of (5), furthermore (6) and (7) are formally exactly the same for DT case. The transfer characteristics of the closed-loop is now

$$y=R_rK_rGy_r+\left(1-R_wK_wG\right)w=R_rG_rG_-z^{-d}y_r++\left(1-R_wG_wG_-z^{-d}\right)w=y_t+y_d \tag{16}$$

Because the optimization of the embedded filters G_r and G_w requires special knowledge and practice of getting the solution from a *DE* and *NP*approximation, suboptimal design is mostly applied assuming $G_r=G_w=1$. In such cases the influence of the invariant process factors are not attenuated at all, so they appear in the closed-loop characteristics (15) directly. Such *G2DOF* control scheme is shown in Fig. 9.

It follows from the above discussion that it is not necessary to apply the classical SMITH predictor principle, instead it is more effective to use the regulator design procedure of the *G2DOF* controller scheme.

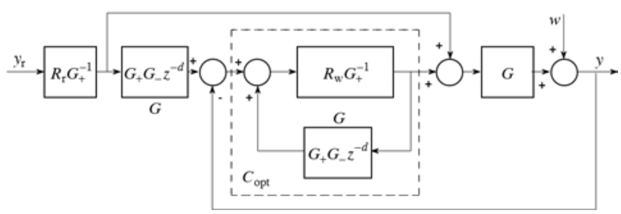

Fig. 9: Discrete-Time *G2DOF*Control System for the Suboptimal $G_r=G_w=1$ case

VI. Simple Examples

Example 1

Consider a very simple first order time-delay process

$$P=\frac{1}{1+10s}e^{-5s}\ ;\ P_+=\frac{1}{1+10s}\ ;\ \overline{P}_-=e^{-5s}\ ;\ P_-=1 \tag{17}$$

The tracking and disturbance rejection reference models are

$$R_r=\frac{1}{1+4s} \quad \text{and} \quad R_w=\frac{1}{1+2s} \tag{18}$$

Here $P_-=1$, therefore $G_r=G_w=1$ is the optimal selection for the embedded filters.

Design a YOULA-paramererized optimal regulator.

$$C_{opt}=\frac{R_wG_wP_+^{-1}}{1-R_wG_wP_-e^{-sT_d}}=\frac{1}{1-R_we^{-sT_d}}R_wP_+^{-1}==\frac{1}{1-\frac{1}{}e^{-5s}}\frac{1+10s}{1+2s}=\frac{\left(1+2s\right)\left(1+10s\right)}{1+2s-e^{-5s}} \tag{19}$$

and the optimal serial compensator is

$$R_rK_r=R_rG_rP_+^{-1}=R_rP_+^{-1}=\frac{1+10s}{1+4s} \tag{20}$$

Both transfer functions are realizable. Because $C_{opt}(s=0)=\infty$ the regulator is integrating obtained from the condition $R_w(s=0)=1$. The optimal final closed-loop is shown in Fig. 10. Although all blocks are realizable in this scheme it is very unrealistic that the real CT models of the true process are applied in a practical application. Here the real difficulty is the realization of the time-delay. So this example stands only to represent the *YP* based *G2DOF* design procedure.

It is easy to check that the closed-loop characteristics is -

$$y_{opt}=R_re^{-sT_d}y_r+\left(1-R_we^{-sT_d}\right)w==\frac{1}{1+4s}e^{-5s}y_r+\left(1-\frac{1}{1+2s}e^{-5s}\right)w \tag{21}$$

according to the general theory.

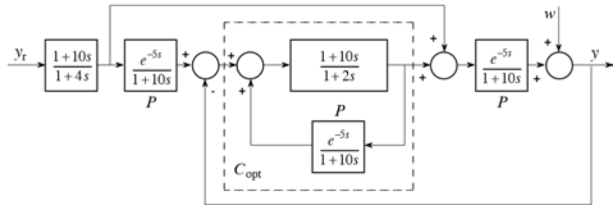

Fig. 10: The Designed Optimal Closed-Loop of the Example

Example 2

Consider the DT model of a very simple first order timedelay process

$$G=\frac{0.2z^{-1}}{1-0.8z^{-1}}z^{-3}=\frac{0.2z^{-4}}{1-0.8z^{-1}}\ ;\ G_+=\frac{0.2z^{-1}}{1-0.8z^{-1}}\qquad \text{and}\qquad G_-=1 \tag{22}$$

It is required to speed up the process by a closed-loop. Design a *YP* controller. Select the reference models

$$R_r=\frac{0.8z^{-1}}{1-0.2z^{-1}}\quad \text{and}\quad R_w=\frac{0.5z^{-1}}{1-0.5z^{-1}} \tag{23}$$

Because $G_-=1$, there is no optimization task, so the selections $G_r=1$ and $G_w=1$ are optimal. The optimal regulator is

$$C_{opt}=\frac{R_wG_wG_+^{-1}}{1-R_wG_wG_-z^{-d}}=\frac{1}{1-R_wz^{-d}}R_wG_+^{-1}==\frac{1}{1-\frac{0.5z^{-1}}{1-0.5z^{-1}}z^{-3}}\frac{0.5z^{-1}}{1-0.5z^{-1}}\frac{1-0.8z^{-1}}{0.2z^{-1}}==\frac{2.5\left(1-0.8z^{-1}\right)}{1-0.5z^{-1}-0.5z^{-4}} \tag{24}$$

and the serial compensator is

$$R_rG_+^{-1}=\frac{0.8z^{-1}}{1-0.2z^{-1}}\frac{1-0.8z^{-1}}{0.2z^{-1}}=\frac{4\left(1-0.8z^{-1}\right)}{1-0.2z^{-1}} \tag{25}$$

The optimal final closed-loop is shown in Fig. 11. Observe that $C_{opt}(z=1)=\infty$, i.e. the regulator is an integrating one, which follows from the condition $R_w(z=1)=1$.

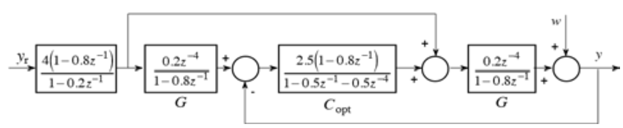

Fig.11: The designed optimal closed-loop of the example

The closed-loop characteristics is

$$y_{opt}=R_rz^{-d}y_r+\left(1-R_wz^{-d}\right)w=\frac{0.8z^{-1}}{1-0.2z^{-1}}z^{-3}y_r++\left(1-\frac{0.5z^{-1}}{1-0.5z^{-1}}z^{-3}\right)w=\frac{0.8z^{-4}}{1-0.2z^{-1}}y_r+\left(1-\frac{0.5z^{-4}}{1-0.5z^{-1}}\right)w \tag{26}$$

which exactly corresponds to our design goals.

This example shows that there is no applicability problem for DT regulator design. These filters are easy to be realized in a computer controlled system.

Example 3.

The continuous first order plant with significant time delay is given by the transfer function

$$P(s)=\frac{1}{1+10s}e^{-30s} \tag{27}$$

The plant is sampled with sampling time $T_s=5$ sec and a zero order hold is applied at its input. Let us design a *PI* controller eensuring about 60° of phase margin, a Smith predictor and a YOULA-parameterized controller. Compare the reference signal tracking and output disturbance rejection behaviour of the three control systems. Demonstrate the effect of time delay mismatch.

The pulse transfer function of the plant is

$$G(z)=\frac{0.3935}{z-0.6065}z^{-6} \quad (28)$$

The pulse transfer function of the *PI* controller [7] applying pole cancellation with a gain ensuring the required phase margin is

$$C_{PI}(z)=0.204\frac{z-0.6065}{z-1} \quad (29)$$

The SMITH predictor controller C_+ is designed for the delay free process as a *PI* controller and it is obtained as

$$C_+(z)=2.5\frac{z-0.6065}{z-1} \quad (30)$$

Then it is transformed to the SMITH predictor form according to the discretized version of (11).

$$C_s(z)=\frac{2.5z^7-1.516z^6}{z^7-0.01636z^6-0.9837} \quad (31)$$

In the case of the YOULAparameterized controller let us choose the disturbance filter

$$R_w(s)=\frac{1}{1+5s} \quad (32)$$

and rhe reference filter as

$$R_r(s)=\frac{1}{1+8s} \quad (33)$$

whose pulse transfer functions are

$$R_w(z)=\frac{0.6321}{z-0.3679} \text{ and } R_r(z)=\frac{0.4647}{z-0.5353}, \quad (34)$$

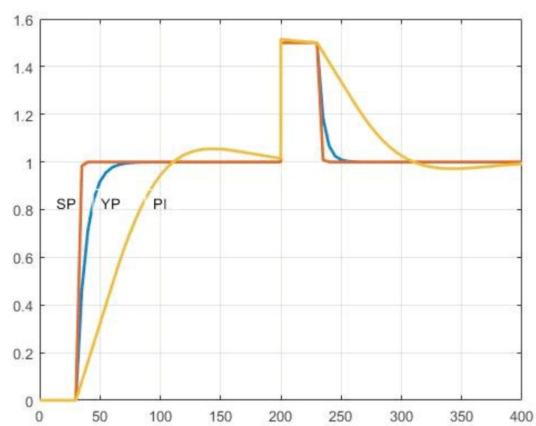

Fig. 12: Step response and disturbance rejection dynamics of the *PI*, SMITH and YOULA controllers

The YOULA parameter supposing $G_r = G_w = 1$ is

$$Q(z)=R_w(z)G_+^{-1}(z)=\frac{0.6321}{z-0.3679}\cdot\frac{z-0.6065}{0.3935} \quad (35)$$

Figure 12 shows the step response and a shifted step disturbance rejection of the three controllers.

It is seen that in case of significant time delay SMITH predictor and the YOULA parameterized controllers ensure significant acceleration compared to the *PI* controller.

Figure 13 demonstrates the effect of time delay mismatch in the case of the SMITH and the YOULA controllers. The time delay of the model is 30, while the time delay of the process is 33.

It is seen that the YOULA parameterized controller tolerates much better the inaccuracy of the parameter than the SMITH predictor. While the SMITH predictor is very sensitive to the inaccuracies in the parameters (it is not robust), the filters in the YOULA parameterized controller can be designed for robust behaviour [11].

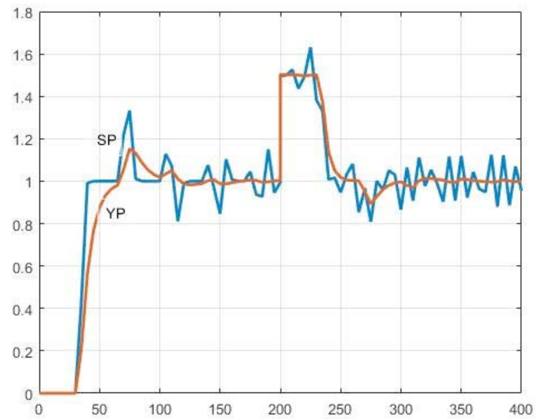

Fig. 13: The Effect of Time Delay Mismatch in Case of the SMITH and the YOULA Controllers

These are, of course, very simple examples standing only to present the simplicity of the *G2DOF* controller scheme, which should replace the classical approach of a SMITH predictor.

VII. Conclusions

The SMITH predictor is a classical method of handling time-delay in closed-loop control design. It is shown that this method is a subclass of the *YP* based *G2DOF* control scheme. An obvious drawback of the SMITH predictor is that the closed-loop properties can not be designed directly using simple algebraic methods, which is possible in the *G2DOF* structure. The *G2DOF* scheme allows even the optimal attenuation of the invariant process factors. The appropriate choice and design of the filters allows to influence such important properties as performance and robustness. So the paper suggests to use the newer methodology to design DT controllers for time-delay processes.

The role of the SMITH predictor remains important in the history of control engineering, because it was one of the first, easy to use and widely applied method to simply eliminate the influence of the delay in the design of closed-loop control properties. Nevertheless this method is sensitive to the accurate knowledge of the time delay.

The recent theoretical developments and easily applicable algebraic design methods allow to use more effective and more general controller design procedures.

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