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The Reducibility of Modal Syllogisms based on the Syllogism EI+O-2

By Long Wei & Xiaojun Zhang

Anhui University

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In other words, there are reducible relations between the modal syllogism □EI+O-2 and the other 38 valid modal syllogisms. There are infinitely many instances in natural language corresponding to any valid modal syllogism. Therefore, this study has theoretical value and practical significance for natural language information processing in computer science.

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The Reducibility of Modal Syllogisms based on the Syllogism □EI+O-2

Long Wei [α] & Xiaojun Zhang [σ]

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I. Introduction

Syllogistic reasoning plays a crucial part in natural language information processing (Long, 2023). Various common syllogisms have been researched and discussed, including generalized syllogisms (Murinov and Novak, 2012), Aristotelian syllogisms (Hui, 2023), Aristotelian modal syllogisms (Cheng, 2023), and so on. In this paper, we restrict our attention to the reducibility of Aristotelian modal syllogisms (Xiaojun, 2018).

Some scholars such as Łukasiewicz (1957), Triker (1994), Nortmann (1996) and Brennan (1997) believed that it is almost impossible to find consistent formal models for Aristotelian modal syllogistic. Smith (1995) summarized the previous researches and proposed that Aristotelian modal syllogistic is incoherent. This view is still prevailing today. In view of this situation, this article attempts to explore a consistent interpretation for Aristotelian modal syllogistic. Specifically, this paper firstly proves the validity of the syllogism □EI+O-2, and then take this syllogism as the basic axiom to derive the other 38 valid modal syllogisms according to modern modal logic and generalized quantifier theory.

Author α σ: School of Philosophy, Anhui University, Hefei, China.
e-mails: 657703460@qq.com, 591551032@qq.com

II. Preliminaries

In this article, it is convenient to represent the lexical variables by capital letters *P*, *M* and *S*, the universe of lexical variables by *D,* any one of the four Aristotelian quantifiers (i.e. *all*, *no*, *some* and *not all*) by *Q.* For Aristotelian syllogisms, there are four types of sentences including 'All *P* are *M*', 'No *P* are *M*', 'Some *P* are *M*' and 'Not all *P* are *M*'. They are abbreviated as the proposition A, E, I and O respectively. An Aristotelian modal syllogism can be obtained by adding one to three non-overlapping necessary operator (i.e.■) or/and possible operator (i.e.+) to an Aristotelian syllogism.

For example, an Aristotelian modal syllogism can be described as the following.

Major premise: No women are necessarily NBA players.

Minor premise: Some millionaires are NBA players.

Conclusion: Not all millionaires are possibly women.

Let *P* be the set of all the women in the universe, *M* be the set of all the NBA players in the universe, and *S* be the set of all the millionaires in the universe. Therefore, this example can be formalized by ■*no*(*P*, *M*)→(*some*(*S*, *M*)→+*not all* (*S*, *P*)), whose abbreviation is□EI +O-2, similarly to other Aristotelian modal syllogisms.

The following definitions, facts and rules can be obtained from modal logic (Chellas, 1980) and generalized quantifier theory (Peters and Westerståhl, 2006). For the sake of convenience, 'if and only if' is abbreviated as 'iff'.

Definition 1:

1. *All* (*P, M*) is true iff $P\subseteq M$ is true.
2. ■*all* (*P, M*) is true iff $P\subseteq M$ is true in any possible world.
3. +*all* (*P, M*) is true iff $P\subseteq M$ is true in at least one possible world.
4. *No* (*P, M*) is true iff $P\cap M=\emptyset$ is true.
5. ■*no* (*P, M*) is true iff $P\cap M=\emptyset$ is true in any possible world.
6. +*no* (*P, M*) is true iff $P\cap M=\emptyset$ is true in at least one possible world.
7. *some* (*P, M*) is true iff $P\cap M\neq\emptyset$ is true.
8. ■*some* (*P, M*) is true iff $P\cap M\neq\emptyset$ is true in any possible world.

9. +*some* (*P*, *M*) is true iff *P*∩*M*≠∅ is true in at least one possible world.
10. *not all* (*P*, *M*) is true iff *P*⊈*M* is true.
11. ■*not all* (*P*, *M*) is true iff *P*⊈*M* is true in any possible world.
12. +*not all* (*P*, *M*) is true iff *P*⊈*M* is true in at least one possible world.

Definition 2: Q¬ (*P*, *M*) $=_{def}$ *Q* (*P*, *D*−*M*).

Definition 3: ¬Q (*P*, *M*) $=_{def}$ It is not that *Q* (*P*, *M*).

The following Fact 1 to Fact 4 are the basic knowledge in generalized quantifier theory, so it is reasonable to omit the proofs of them here.

Fact 1: (1) *some* (*P*, *M*)↔*some* (*M*, *P*);
(2) *no* (*P*, *M*) ↔*no*(*M*, *P*).

Fact 2: (1) *all* (*P*, *M*)=*no*¬(*P*, *M*);
(2) *no* (*P*, *M*)=*all*¬(*P*, *M*);
(3) *some* (*P*, *M*)=*not all*¬(*P*, *M*);
(4) *not all* (*P*, *M*)=*some*¬(*P*, *M*).

Fact 3: (1) ¬*all* (*P*, *M*)=*not all* (*P*, *M*);
(2) ¬*no* (*P*, *M*)=*some* (*P*, *M*);
(3) ¬*some* (*P*, *M*)=*no* (*P*, *M*);
(4) ¬*not all* (*P*, *M*)=*all* (*P*, *M*).

Fact 4: (1) ⊢*all* (*P*, *M*)→*some* (*P*, *M*);
(2) ⊢*no* (*P*, *M*)→*not all* (*P*, *M*).

According to modal logic (Chellas, 1980), + is definable in terms of ¬ and ■, that is to say that ■*Q*(*P*, *M*)□ ↔¬+¬*Q*(*P*, *M*) and +*Q*(*P*, *M*) ↔¬■¬*Q*(*P*, *M*) hold at every possible world. The following Fact 5 to Fact 8 can be proved by modal logic (Chagrov and Zakharyaschev, 1997).

Fact 5: (1) ¬■*Q* (*P*, *M*)=+¬*Q* (*P*, *M*);
(2) ¬+*Q*(*P*, *M*) =■¬*Q* (*P*, *M*).

Fact 6: ⊢■*Q* (*P*, *M*)→*Q* (*P*, *M*).

Fact 7: ⊢*Q* (*P*, *M*)→+*Q* (*P*, *M*).

Fact 8: ⊢■*Q* (*P*, *M*)→+*Q* (*P*, *M*).

The following rules in first order logic can be applied to Aristotelian syllogistic and Aristotelian modal syllogistic, in which *p*, *q*, *r* and *s* represent propositional variables.

Rule 1: (Subsequent weakening): From ⊢(*p*→(*q*→*r*)) *and* ⊢(*r*→*s*) infer ⊢(*p*→(*q*→*s*)).

Rule 2: (anti-syllogism): From ⊢(*p*→(*q*→*r*)) infer ⊢(¬*r*→(*p*→¬*q*)) or ⊢(¬*r*→(*q*→¬*p*)).

III. Reduction Between the Syllogism □EI+O-2 and the Other 38 Modal Syllogisms

Theorem 1 means that the syllogism □EI+O-2 is valid. The following theorems from Theorem 2 to Theorem 9 demonstrate that there are reducible relations between the syllogism□EI +O-2 and the other 38 valid modal syllogisms. For example, '(2.1) □EI+O-2⇒□E■AE- 1' in Theorem 2 means that the validity of syllogism ■E■AE-1 can be derived from the validity of □EI+O-2. This sheds light on the reducibility between the two syllogisms. Other cases are similar.

Theorem 1 (□EI+O-2): ■*no*(*P*, *M*)→(*some*(*S*, *M*)→+*not all*(*S*, *P*)) is valid.

Proof: The syllogism □EI +O-2 is the abbreviation of the second figure syllogism ■*no*(*P*, *M*)→(*some*(*S*, *M*)→ +*not all*(*S*, *P*)). Suppose that +*no*(*P*, *M*) and *some*(*S*, *M*) are true, then *P*∩*M*=ϕ is true at any possible world in terms of the clause (5) in Definition 1, and *S*∩*M*≠ϕ is true in terms of the clause (7) in Definition 1. Now it is clear that *S*⊈*P* is true in at least one possible world. Therefore, +*not all*(*S*, *P*) is true according to the clause (12) in Definition 1. It indicates the validity of ■*no*(*P*, *M*)→(*some*(*S*, *M*)→+*not all*(*S*, *P*)), just as desired.

Theorem 2: The validity of the following two syllogisms can be inferred from □EI+O-2 :

(2.1) □EI+O-2⇒■E■AE-1

(2.2) □EI+O-2⇒I □ A+I-3

Proof: For (2.1). In line with Theorem 1, it follows that □EI+O-2 is valid, and its expansion is that ■*no*(*P*, *M*)→(*some*(*S*, *M*)→+*not all*(*S*, *P*)). And then it can be derived that ¬+*not all*(*S*, *P*) →(■*no*(*P*, *M*)→¬*some*(*S*, *M*)) in the light of Rule 2. According to Fact 5, what is obtained is that ■¬*not all*(*S*, *P*)→(■*no*(*P*, *M*)→¬*some* (*S*, *M*)). One can obtain that ¬*not all*(*S*, *P*)=*all*(*S*, *P*) and ¬*some*(*S*, *M*)=*no*(*S*, *M*) on the basis of the clause (4) and (3) in Fact 3. Therefore, it can be seen that ■*all*(*S*, *P*)→(■*no*(*P*, *M*)→*no*(*S*, *M*)) is valid. That is to say that ■E■AE-1 can be deduced from □EI+O-2, as desired. The proof of (2.2) is similar to that of (2.1).

Theorem 3: The validity of the following four syllogisms can be inferred from □EI+O-2:

(3.1) □EI+O-2⇒ □ EI+O-1

(3.2) □EI+O-2⇒■E■AE-1⇒■E■AE-2

(3.3) □EI+O-2⇒■E■AE-1⇒■A■EE-4

(3.4) □EI+O-2⇒■E■AE-1⇒■A■EE-4⇒■A■EE-2

Proof: For (3.1). According to Theorem 1, it follows that □EI+O-2 is valid, and its expansion is that ■*no*(*P*, *M*)→(*some*(*S*, *M*)→+*not all*(*S*, *P*)). In line with the clause (2) in Fact 1, it can be seen that □*no*(*P*, *M*) ↔□*no*(*M*, *P*). Therefore, it can be seen that □*no*(*M*, *P*)→(*some*(*S*, *M*)→ +*not all*(*S*, *P*)), i.e.□EI +O-1 can be deduced from □EI+O-2. The proofs of the other cases are along similar lines to that of (3.1).

Theorem 4: The validity of the following four syllogisms can be inferred from □EI+O-2:

(4.1) □EI+O-2⇒■E■AE-1⇒■E■AO-1

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(4.2) □EI+O-2⇒■E■AE-1⇒■E■AE-2⇒■E■AO-2

(4.3) □EI+O-2⇒■E■AE-1⇒■A■EE-4⇒■A■EO-4

(4.4) □EI+O-2⇒■E■AE-1⇒■A■EE-4⇒■A■EE-2⇒■A■EO-2

Proof: For (4.1). According to (2.1) □EI+O-2⇒■E■AE-1, it follows that ■E■AE-1 is valid, and its expansion is that ■*no*(*P, M*)→(■*all*(*S, P*)→*no*(*S, M*)). It can be seen that ⊢ *no*(*Y, X*) →*not all*(*Y, X*), using the clause (2) in Fact 4. Hence, ■*no*(*P, M*)→(□*all*(*S, P*)→*not all*(*S, M*)) is valid by means of Rule 1. In other words, ■E■AO-1 can be derived from □EI+O-2. The other cases can be similarly demonstrated.

Theorem 5: The validity of the following two syllogisms can be inferred from □EI+O-2:

(5.1) □EI+O-2⇒□AO+O-2

(5.2) □EI+O-2⇒■E■AE-1⇒■A■AA-1

Proof: For (5.1). In line with Theorem 1, it follows that □EI+O-2 is valid, and its expansion is that +■*no*(*P, M*)→(*some*(*S, M*)→+*not all*(*S, P*)). It is clear that *no*(*P, M*)=*all*¬(*P, M*) and *some*(*S, M*)=*not all*¬(*S, M*) hold on the basis of the clause (2) and (3) in Fact 2. Then one can infer that□ *all*¬(*P, M*)→(*not all*¬(*S, M*)→+*not all*(*S, P*)). It can be seen that *all*¬(*P, M*)=*all*(*P, D−M*) and *not all*¬(*S, M*)=*not all*(*S, D–M*) according to Definition 2. Hence, the validity□ of *all*(*P, D-M*)→ (*not all*(*S, D−M*)→+*not all*(*S, P*)) is straightforward. That is to say that□AO +O-2 can be deduced from□EI +O-2, as desired. The proof of (5.2) is along a similar line to that of (5.1).

Theorem 6: The validity of the following six syllogisms can be inferred from □EI+O-2:

(6.1) □EI+O-2⇒■E■AE-1⇒■A■AA-1⇒■A■AI-1

(6.2) □EI+O-2⇒■E■AE-1⇒■A■AA-1⇒■A■AI-1⇒■A■AI-4

(6.3) □EI+O-2⇒□EI+O4

(6.4) □EI+O-2⇒I□ A+I-3⇒□AI+I-3

(6.5) □EI+O-2⇒I□A+I-3⇒□AI+I-3⇒I□ A+I-4

(6.6) □EI+O-2⇒I□A+I-3 ⇒ □AI+I-3⇒ □AI+I-1

Proof: For (6.1). In line with (5.2) □EI +O-2⇒■E■AE-1⇒■A■AA-1, it follows that ■A■AA-1 is valid, and its expansion is that ■*all*(*P, M*)→(■*all*(*S, P*)→*all*(*S, M*)). Then, it can be seen that *all*(*S, M*)→*some*(*S, M*) according to the clause (1) in Fact 4. Hence, it can be proved that ■*all*(*P, M*)→(■*all*(*S, P*)→*some*(*S, M*)) is valid. In other words, the syllogism ■A■AI-1 can be derived from □EI+O-2.

For (6.2). According to (6.1) □EI+O-2⇒■E■AE-1⇒■A■AA-1⇒■A■AI-1, it follows that ■A■AI-1 is valid, and its expansion is that ■*all*(*P, M*)→(■*all*(*S, P*)→*some* (*S, M*)). Then, what is obtained is that ■*some*(*S, M*)↔ ■*some*(*M, S*), using the clause (1) in Fact 1. It is reasonable to say that ■*all*(*P, M*)→(■*all* (*S, P*)→ ■*some*(*M, S*)) is valid. That is to say that the syllogism ■A■AI-4 can be derived from ■A■AI-1. The proofs of other cases are along similar lines to that of (6.2).

Theorem 7: The validity of the following five syllogisms can be inferred from □EI◇O-2:

(7.1) □EI+O-2⇒■E■AE-1⇒■A■AA-1⇒O□A+O-3

(7.2) □EI+O-2⇒■E■AE-1⇒■E■AE-2⇒■E■AO-2⇒□AA+ I-3

(7.3) □EI+O-2⇒■E■AE-1⇒■A■EE-4⇒■A■EO-4⇒□EA+ O-4

(7.4) □EI+O-2⇒■E■AE-1⇒■A■AA-1⇒■A■AI-1⇒■AE+ O-2

(7.5) □EI+O-2⇒■E■AE-1⇒■A■AA-1⇒■A■AI-1⇒■AE+ O-2⇒E■A+O-3

Proof: For (7.1). In line with (5.2) □EI+O-2⇒E■AE-1⇒■A■AA-1, it follows that ■A■AA-1 is valid, whose expansion is that ☑*all*(*P, M*)→(■*all*(*S, P*)→*all*(*S, M*)). And then it can be derived that ¬*all*(*S, M*)→(■*all*(*S, P*)→¬■*all*(*P, M*)) in the light of Rule 2. Thus one can obtain that ¬*all*(*S, M*)→(■*all*(*S, P*)→+¬*all*(*P, M*)) according to Fact 5. It is clear that ¬*all*(*S, M*)=*not all*(*S, M*) and ¬*all*(*P, M*)=*not all*(*P, M*) based on the clause (1) in Fact 3. Therefore, it can be seen that *not all*(*S, M*)→(■*all*(*S, P*)→ +*not all*(*P, M*)) is valid. That is to say that O□A+O-3 can be deduced from □EI+O-2. The proofs of other cases follow the similar pattern as that of (7.1).

Theorem 8: The validity of the following four syllogisms can be inferred from □EI+O-2:

(8.1) □EI+O-2⇒□EI+O-4⇒□EI+O-3

(8.2) □EI+O-2⇒■E■AE-1⇒■A■EE-4⇒■A■EO-4⇒□EA+ O-4⇒□EA+O-3

(8.3) □EI+O-2⇒■E■AE-1⇒■A■AA-1⇒■A■AI-1⇒■AE+ O-2⇒■AE+O-4

(8.4) □EI+O-2⇒■E■AE-1⇒■A■AA-1⇒■A■AI-1⇒■AE+ O-2⇒E■A+O-3⇒E■A+O-4

Proof: For (8.1). In line with (6.3) □EI+O-2⇒□EI+O-4, it follows that □EI+O-4 is valid, and its expansion is that ■*no*(*P, M*)→(*some*(*M, S*)→ +*not all*(*S, P*)). Then, what is obtained is ■*no*(*P, M*)↔ ■*no*(*M, P*), using the clause (2) in Fact 1. Hence, it can be proved that ■*no*(*M, P*) →(*some*(*M, S*)→ +*not all*(*S, P*)) is valid, i.e. the syllogism □EI+O-3 can be derived from □EI+O-2. The other cases can be similarly proved.

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Theorem 9: The validity of the following eleven syllogisms can be inferred from ▯EI+O-2:

(9.1) ▯EI+O-2⇒■E■AE-1⇒■E■A+E-1

(9.2) ▯EI+O-2⇒■E■AE-1⇒■E■AE-2⇒■E■A+E-2

(9.3) ▯EI+O-2⇒■E■AE-1⇒■A■EE-4⇒■A■E+E-4

(9.4) ▯EI+O-2⇒■E■AE-1⇒■A■EE-4⇒■A■EE-2⇒■A■E+E-2

(9.5) ▯EI+O-2⇒■E■AE-1⇒■E■AO-1⇒■E■A+O-1

(9.6) ▯EI+O-2⇒■E■AE-1⇒■E■AE-2⇒■E■AO-2⇒■E■A+O-2

(9.7) ▯EI+O-2⇒■E■AE-1⇒■A■EE-4⇒■A■EO-4⇒■A■E+O-4

(9.8) ▯EI+O-2⇒■E■AE-1⇒■A■EE-4⇒■A■EE-2⇒■A■E O-2⇒■A■E+O-2

(9.9) ▯EI+O-2⇒■E■AE-1⇒■A■AA-1⇒▯A■A+A-1

(9.10) ▯EI+O-2⇒■E■AE-1⇒■A■AA-1⇒■A■AI-1⇒■A■A+I-1

(9.11) ▯EI+O-2⇒■E■AE-1⇒■A■AA-1⇒■A■AI-1⇒■A■AI-4⇒■A■A+I-4

Proof: For (9.1). In line with (2.1) ▯EI+O-2⇒■E■AE-1, it follows that ■E■AE-1 is valid. It is clear that E⇒+E according to Fact 7. Therefore, the validity of ■E■A+E-1 is straightforward. The proofs of other cases follow the same pattern as that of (9.1).

So far, the other 38 valid Aristotelian modal syllogisms have been derived from the validity of the syllogism ▯EI+O-2 on the basis of modern modal logic and generalized quantifier theory.

IV. Conclusion and Future Work

This paper firstly demonstrates the validity of the syllogism ▯EI+O-2, and then takes it as the basic axiom to derive the other 38 valid modal syllogisms by taking advantage of some reasoning rules in classical propositional logic, the symmetry of two Aristotelian quantifiers (i.e. *some* and *no*), the transformation between an Aristotelian quantifier and its three negative quantifiers, and some facts in first order logic. In other words, there are reducibility between the syllogism ▯EI+O-2 and the other 38 valid Aristotelian modal syllogisms. Moreover, the above deductions may provide a consistent interpretation for Aristotelian modal syllogistic. There are infinitely many instances in natural language corresponding to any valid modal syllogism. Therefore, this study has significant theoretical value and practical significance to natural language information processing in computer science.

Can the remaining valid Aristotelian modal syllogisms be derived from a few valid modal syllogisms (such as □E▯I□O-2, □E□I◇O-2, □E◇I◇O-2, □E□IO-2, ◇EI◇O-2, E◇I◇O-2, □EI◇O-2, E□I◇O-2, □EIO-2, E□IO-2 and EI◇O-2), and how to construct a coherent formal system for Aristotelian modal syllogistic? These questions need to be explored in depth.

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