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# System of Linear Equations, Guassian Elimination

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*Keywords :* *direct, indirect, backward stage, forward stage.*

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# System of Linear Equations, Guassian Elimination

Suriya Gharib [α], Syeda Roshana Ali [σ], Rabia Khan [ρ], Nargis Munir [ω] & Memoona khanam [¥]

*Abstract-* In this paper linear equations are discussed in detail along with elimination method. Guassian elimination and Guass Jordan schemes are carried out to solve the linear system of equation. This paper comprises of matrix introduction, and the direct methods for linear equations. The goal of this research was to analyze different elimination techniques of linear equations and measure the performance of Guassian elimination and Guass Jordan method, in order to find their relative importance and advantage in the field of symbolic and numeric computation. The purpose of this research is to revise an introductory concept of linear equations, matrix theorey and forms of Guassian elimination through which the performance of Guass Jordan and Guassian elimination can be measured.

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## I. Introduction

A system of equation is a set or collection of equations solved together. Collection of linear equations is termed as system of linear equations. They are often based on same set of variables. Various methods have been evolved to solve the linear equations but there is no best method yet proposed for solving system of linear equations[1]. Various methods are proposed by different mathematicians based on the speed and accuracy. However speed is an important factor for solving linear equations where volume of computation is so large. Linear equation methods are divided into two categories. Direct and Indirect. Each category comprises of several elimination methods used for solving equations. this paper deals with Guassian elimination method, a direct method for solving system of linear equations. An introductory portion of Guass Jordan elimination is also carried out in order to analyze the performance of both methods. Indirect methods are basically iterative methods and these methods have an advantage in a sense that they require fewer multiplication steps for large computations. Iterative methods can be implemented in smaller programs and are fast enough. With study of system of linear equation one must be familiar with matrix theory that howdifferent operations are performed on a desired matrix to calculate the result.

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## II. History of Guassian Elimination[2]

Classic books on the History of Mathematics, as well as recent studies on this subject, place the origins of Guassian Elimination in a variety of ancient texts from different places and times: China, Greece, Rome, India, medieval Arabic countries, and European Renaissance. However, it is not exact to say that these ancient texts describe what we understand today as the method of Guassian Elimination, since these texts mainly present some specific problems that are solved in a way that is accepted as Guassian Elimination, butthey do not include any explicit statement of the set of rules that constitute the method of GE. The schoolbook elimination period corresponds to the development of GE essentially as it is presented incurrent high school textbooks. This period started with Isaac Newton who lectured on Algebraas it appeared in Renaissance texts.

Isaac Newton established first the rules of Gaussian elimination as they are still presented in current high school textbooks. Carl Friedrich Gauss developed efficient methods for solving normal equations, i.e., the special type of linear equations that may arise in solutions of least square problems, via Gaussian elimination of linear equations via hand computations.

## III. Guassian Elimination

Guassian elimination is the standard method for solving linear equations. As it is a ubiquitous algorithm and plays a fundamental role in scientific computation. Guassian elimination is a tool for obtaining the solution of equations, to compute the determinant, for deducing rank of coefficient matrix. However Guassian Elimination depends more on matrix analysis and computation. It emphasis on block pivoting, methods of iteration and a means to improve the computed solution quality. It involves two stages forward and backward stage.

*Forward stage:* Unknowns are eliminated in this stage by manipulation of equations and constitute an echelon form.

*Backward stage:* it is related with back substitution process on the reduced upper triangular method resulting in a solution of equation.

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*a) Steps to Solve Guassian Elimination*

Guassian Eliminationis systematic application of elementary row operations in system of equations [2]. It converts the linear system of equations to upper triangular form, from which solution of equation is determined. Guassian elimination is summarized in the above mentioned steps[3]:

i. Augmented matrix must be written for the system of linear equations..
ii. Transform A to upper triangular form using row operations on{A/b}. diagonal elements may not be zero.
iii. Use back substitution for finding the solution of problem.

-Consider the system of linear equations with involving n variables[3].

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = a_1,n+1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = a_2,n+1$$
$$a_{31}x_1 + a_{32}x_2 + \ldots + a_{3n}x_n = a_3,n+1$$
$$\ldots \ldots \ldots \ldots \ldots \ldots$$
$$a_{n1}x_1 + a_{n3}x_2 + \ldots + a_{nn}x_n = a_n,n+1$$

Where $a_{ij}$ and $a_{i,j+1}$are constants, xi's are variables. The system becomes equal to:
AX=B

$$\begin{bmatrix} a_{11} a_{12} a_{13} \ldots a_{1n} \\ a_{21}\ a_{22} a_{23} \ldots a_{2n} \\ a_{31} a_{32} a_{33} \ldots a_{3n} \\ : \quad : \quad : \\ a_{1n} a_{2n} a_{3n} \ldots a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ : \\ x_n \end{bmatrix} = \begin{bmatrix} a_{1,n+1} \\ a_{2,n+1} \\ a_{3,n+1} \\ : \\ a_{n,n+1} \end{bmatrix}$$

*Step 1:* Store the coefficients in an augmented matrix. The superscript on $a_{ij}$ means that this is the first time that a number is stored in location (i, j).

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} a_{13} \ldots a_{1n} & & & a_{1,n+1} \\ a_{21} & a_{22} a_{23} \ldots a_{2n} & & & a_{2,n+1} \\ a_{31} & a_{32} a_{33} \ldots a_{3n} & & & a_{3,n+1} \\ : & : & & & : \\ a_{1n} & a_{2n} & a_{3n} & \ldots\ a_{nn} & a_{n,n+1} \end{array}\right]$$

*Step 2 :* If necessary, shift rows so that a≠10 , theneliminate x1 in row2 through n .The new elements are written aij to indicate that this is thesecond time that a number has been stored in the matrix atlocation ( i , j ).

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} a_{13} \ldots a_{1n} & & & a_{1,n+1} \\ 0 & a_{22} a_{23} \ldots a_{2n} & & & a_{2,n+1} \\ 0 & a_{32} a_{33} \ldots a_{3n} & & & a_{3,n+1} \\ : & : & & & : \\ 0 & a_{2n} & a_{3n} & \ldots\ a_{nn} & a_{n,n+1} \end{array}\right]$$

*Step 3 :* New elements are written aij indicate that this is thethird time that a number has been stored in the matrix at location ( i , j ) .

$$\left[\begin{array}{cc|c} a_{11} & a_{12} a_{13} \ldots a_{1n} & a_{1,n+1} \\ 0 & 0\ a_{23} \ldots a_{2n} & a_{2,n+1} \\ 0 & 0\ a_{33} \ldots a_{3n} & a_{3,n+1} \\ : & : & : \\ 0 & 0\ a_{3n}\ \ldots\ a_{nn} & a_{n,n+1} \end{array}\right]$$

Final result after the row operation may result in above form:

$$\left[\begin{array}{cc|c} a_{11} & a_{12} a_{13} \ldots a_{1n} & a_{1,n+1} \\ 0 & 0\ a_{23} \ldots a_{2n} & a_{2,n+1} \\ 0 & 0\ a_{33} \ldots a_{3n} & a_{3,n+1} \\ : & : & : \\ 0 & 0\quad 0 \ldots a_{nn} & a_{n,n+1} \end{array}\right]$$

*b) Sequential Algorithm – Gauss Elimination Method[4]*

*Input :* Given Matrix a[1 : n, 1: n+1]

*Output :* x[1 : n]

1. for k = 1 to n-1
2. for i = k+1 to n
3. u = $a_{ik}/a_{kk}$
4. for j = k to n+1
5. aij = $a_{ij}$ – u * $ak_j$
6. next j
7. next i
8. next k
9. $x_n$ = $a_n$,n+1/ann
10. for i = n to 1 step -1
11. sum = 0
12. for j = i+1 to n
13. sum = sum + $a_{ij}$ * $x_j$
14. next j
15. xi = ( $a_i$,n+1 - sum )/$a_{ii}$
16. next i
17. end

*c) Guassian Elimination Through Partial Pivoting*

In actual computational practice, it is necessary to permute the rows of the matrix A (equivalently, the equations of the system Ax = b) for obtaining a reliable algorithm.

The permutations are performed on line as GE proceeds and several permutation (or pivoting). Partial pivoting involves the following steps:

*Step 1:* Select the equation having the larger 1st coefficient in system of equation and place it at the 1st entity of matrix.

$$\begin{bmatrix} 2 & 3 & -1 & 1 \\ 2 & 5 & 1 & 3 \\ 6 & 4 & 1 & 9 \end{bmatrix} > \begin{bmatrix} 6 & 4 & 1 & 9 \\ 2 & 5 & 1 & 3 \\ 2 & 3 & -1 & 1 \end{bmatrix}$$

*Step 2 :* Now perform the elementary row operations and convert the matrix into upper triangular form by using the pivot equation. The resultant matrix after operations may result in the form:

$$\begin{bmatrix} 6 & 4 & 1 & 9 \\ 0 & 5 & 1 & 3 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

*Step 3 :* Make the equation equal to the number of variables and determine the solution of equation.

*d) LU Factorization Guassian Elimination*

LU factorization is the most important mathematical concept used in Guassian Elimination method. It plays a key role in the implementation of GE in modern computers, and, finally, it is essential to facilitate the rounding error analysis of the algorithm. LU factorization method is performed in three steps [5]:

i. **A=LU**, compute the LU factorization.

ii. For **y** solve the lower triangular matrix as **Ly=b** by using forward substitution method(i.e, start by computing the first unknown as **y1=b1** from the first equation, after that compute the second unknown using the value of previous variable and so on.

iii. Compute **x** for the upper triangular matrix using the relation **Ux=y** by using backward substitution method.

Consider the following matrix of the form Ax=b :

$$A=\begin{bmatrix} a_{11} a_{12} a_{13} a_{14} \\ a_{21} a_{22} a_{23} a_{24} \\ a_{31} a_{32} a_{33} a_{34} \\ a_{41} a_{42} a_{43} a_{44} \end{bmatrix}, \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, b=\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

Transform the matrix A into lower and upper triangular matrix for the further computations. Let lower triangular matrix be L and upper triangular matrix be U.

$$L=\begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

$$U=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}$$

From these upper and lower triangular matrix perform the computations for both equations listed in step ii. Calculate the values of unknown.

## IV. Guassian Elimination as Computational Pardigm[6]

Sparse Guassian Elimination was studied in the early 70's. For the vertex elimination on the undirected graphs a graph model was proposed. The structural properties of the vertices has been a major research in the last decades. Also work on optimal elimination tree was carried out, which proved of no importance in sparsity preserving elimination trees, looking towards optimal elimination trees could result in non linear fill. There has been minimum use of tree related graphs elimination outside the sparse Guassian Elimination. Cholesky factor is described in terms of different set of vertices: sets of predecessor and successor, chain elimination and elimination sets. The model of Guassian Elimination gives a precise description of interaction between master, sub problems which are hidden in formulation of dynamic programming. In case of solving blocked linear equations with PDS matrices, proposed model of computation is a straightforward extension of Guassian Elimination (point wise).

Transformation associated with elimination of vertex is simply block elimination using submatrix of block diagonal as block pivot.

One application regarding computational model is in the context of solution of asymmetric blocked structural system of linear system of equations which demonstrates an indirect use in process of solution, rules of assignments of columns to block for block elimination process. These rules provide a new concept of pivoting. Consider zero subcolumn in original data and non zero in partially reduced matrix. Computed subcolumn remains in the column space of some subcolumn in the original data. Substituting one of these subcolumn to the considered subcolumn is appealing. It is complementary to sparse preserving elimination. Cholesky Factorization method is motivated by solution of so called normal equations that come from linearalized KKT system in the context of Newton method.

While considering interior point for solving large scale block problems, from a numerical point of view to solve linear system of equations is of great consideration. Smallest height elimination trees tend to have maximum number of leaves. Block Cholesky includes block LU factorizations, the coefficients of submatrix that correspond to the leaves is the original data. Incomplete factorization consists of factorization from leaves up to the level where data is to be transformed several times by preceding block eliminations.

## V. Algorithms

*a) Guassian Elimination[7]*

1. {*begin Reduction to Triangular form*}

for i= 1 to N- 1 do

for k = i + 1 to N do

$a_{ki} = fl(a_{ki}/a_{11})$

for j = i + 1 to N+ 1 do

$a_{kj} = fl(a_{kj} - a_{ki} * a_{ij})$

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2. { *begin back-Substitution*}

$x_N = fl(a_{N,N+1}/ a_{NN})$

for i= N-1 downto **1** do

for j = N downto $i$ + **1**do

$a_{i,N+1} = fl(a_{1,N+1}\text{-}a_{ij}{*}x_j)$

$x_i = fl(a_{1,N+1}/a_{ij})$

*b) Guass Jordan*

1. {*begin Reduction to Diagonal Form*}

fori = 1 to Ndo

fork = 1 to N(except i ) do

forj= i + 1 to N+1 do

$a_{ki} = fl(a_{ki}/a_{ij})$

for j= i+1 to N+1 do

$a_{kj} = fl(a_{kj}\text{-}a_{kj}{*}a_{ij})$

2. {*beginSolving Diagonal System*}

fori = 1 to N do

## VI. Performance Comparison of Guassian Elimination with Guass Jordan

Guassian elimination and Guass Jordan methods are compared and analyzed on the basis of execution time explained in the following table

| No of variables | Time of Guassian Elimination (millisecond) | Time of Guass Jordan (millisecond) |
|---|---|---|
| 2 | 14 | 25 |
| 3 | 16 | 31 |
| 4 | 20 | 36 |
| 5 | 26 | 39 |
| 6 | 29 | 56 |
| 7 | 46 | 76 |

*Comparison through execution time*

From the above mentioned results its clear that Guassian elimination is more faster than Guass Jordan method. Therefore, an efficient technique for solving linear system of equations, determining the values of unknowns in less time and less complicated procedure.

## VII. Conclusion

There are different direct and indirect methods which are used to compute the linear system of equations. Guassian Elimination is a type of direct method used to calculate the unknown variables. Many scientific and engineering domains of computation may take the form of linear equations. The equations in this field may contain large number of variables and hence it is important to solve these equations in an efficient manner. This paper comprises of Guassian Elimination method an efficient method to solve these equations. Although the comparison on the basis of execution time is carried out along with the Guass Jordan method and it has been concluded that Guassian Elimination is faster than the other elimination methods and it is used in various scientific fields where large number of computations are performed by elimination of variables. Our future directions are to use and develop the simple and efficient method for non linear system of equations.

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