A Novel Classifier for Digital Angle Modulated Signals

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I. Introduction

The ability to identify the modulation type of an arbitrary noisy signal is desirable for different reasons like signal confirmation, spectrum management [1], electronic support measures (ESM), electronic counter measures (ECM) in warfare [2], selection of appropriate demodulator in intelligent moderns and military threat analysis [3]. In defense applications, the analysis of hostile transmissions is important both for extracting secret messages from communication signals and for implementing counter measures. In spectrum monitoring application, unauthorized transmissions are continuously intercepted and analyzed in a given area and frequency band, to detect the unauthorized ones or deviations in the authorized transmissions and finally deciding on the corrective steps. In either of the above cases and in similar applications, the characteristics of the intercepted signals must be determined and before that the modulation type is to be estimated.

A simple communication signal classifier comprises a bank of demodulators, each designed for only one type of modulation of the received signal [4]. An operator examining the demodulators’ outputs could identify the type of modulation of the received signal.

The obvious disadvantages of manual mode can be alleviated by the machine-based modulation classification techniques. More over such automatic classification techniques are to be invariably used in real time systems in modern warfare, surveillance and in situations where the signals are available only for short durations. Liedtke [5] and Jondral [6] used pattern recognition techniques for Classification of modulated signals which require large amounts of data to train the classifier. Dominguez et.al. [7] and Hagiwara et.al. [8] used histograms of instantaneous envelope and modulation indices respectively for classification purpose. However, none of these have the necessary analytical support. Further, the required carrier-to-noise ratio (CNR) for correct classification is greater than 15 dB. In a recent paper [9], an autoregressive (AR) model applied on the instantaneous frequency has been proposed for the classification of PSK and FSK signals. However, this technique requires a CNR of 15dB or more, and is also limited to binary PSK and binary FSK (i.e. M=2) only. In another recent paper [10], higher order cumulants and moments (up to eighth order) were proposed as features in combination with a support vector machine (SVM) classifier to classify MPSK signals for M=2, 4, 8 along with quadrature amplitude modulated (QAM) signals at as low as CNR of 3 dB. The genetic algorithm (GA) was used here for the optimal design of the classifier. However, the FSK signals were not considered and the computation complexity is very high. In [11] modulation classification based on wavelet and fractional fourier transform was proposed but the technique is limited to 2PSK and 2FSK only.

In this paper, a simple but powerful technique for distinguishing between the digital angle modulated signals, that is between MPSK and MFSK signals is proposed. The method is based on the variance ratio of instantaneous frequency of the received signal passed through a pair of concentric band pass filters. The center frequencies of the filters are automatically determined from the short time fourier transform (STFT) analysis. The analytical expressions for the variance of instantaneous frequency of both MPSK and MFSK signals are derived. The expression for the variance ratio of filter outputs is also analytically obtained and is used as the decision statistic. Extensive simulations to validate the proposed method are carried out and the results are in close agreement with the theoretical predictions.

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The paper is organized as follows. In section II, derivation of analytical expressions of variances is presented. In section III, the expression for the proposed decision statistic is derived. The algorithm for the proposed novel classifier is presented in section IV. In section V, details of simulations and the results are presented. Finally concluding remarks and scope of future work are presented in Section VI.

II. INSTANTANEOUS FREQUENCY OF MFSK AND MPSK SIGNALS

In this section, analytical expressions for instantaneous frequency of a noisy sine wave derived by Rice [12] are adapted to obtain expressions for the statistics of instantaneous frequency of PSK and FSK signals. Consider a noisy sine wave given by

\[ s(t) = \Lambda \cos 2\pi f_c \, t + n(t) \]  

(1)

where \( n(t) \) is a zero mean band pass white noise with one sided power spectral density \( \eta \). The instantaneous phase \( \phi_i(t) \), the instantaneous angular frequency \( \omega_i(t) \) and the instantaneous linear frequency \( f_i(t) \) of \( s(t) \) is given by

\[ \phi_i(t) = \tan^{-1} \left( \frac{\bar{s}(t)}{s(t)} \right), \quad \omega_i(t) = \frac{d\phi_i(t)}{dt}, \quad f_i(t) = \frac{1}{2\pi} \frac{d\phi_i(t)}{dt} \]  

(2)

where \( \bar{s}(t) \) is the Hilbert transform of the signal \( s(t) \). Rice [12] showed that the probability density function of angular frequency \( \omega_i(t) \) is given by

\[ p(\omega_i) = \frac{1}{2} \sqrt{\frac{\sigma_{\omega i}^2}{\sigma_n^2}} (1 + z^2)^{-\frac{3}{2}} e^{-\gamma + \frac{\gamma^2}{2}} \left[ (1+y)l_0 \left( \frac{\gamma}{2} \right) + yl_0 \left( \frac{\gamma}{2} \right) \right] \]  

(3)

where

\[ z^2 = \frac{\sigma_n}{\sigma_{\omega i}} \omega_{\omega i}^2; \quad \gamma = \frac{A^2}{2\eta B}; \quad \text{and} \quad y = \frac{\gamma}{1 + z^2} \]  

(4)

and \( l_0 \) and \( l_1 \) are Bessel functions of first the kind, \( B \) is the bandwidth of an ideal band pass filter, \( \sigma_{\omega i}^2 \) and \( \sigma_n^2 \) the variances of the noise and its time derivative respectively and finally \( \gamma \) is the carrier-to-noise ratio (CNR). At high and moderate CNRs, this density approaches gaussian function with a mean of \( 2\pi f_c \) and variance given by

\[ \sigma_{\omega i}^2 = \frac{\pi^2 \eta B^2}{3A^2} \]  

(5)

which can be obtained in terms of CNR as

\[ \sigma_{\omega i}^2 = \frac{\pi^2 B^2}{6\gamma} \]  

(6)

It is easy to see that the mean and variance of instantaneous linear frequency can be respectively written as

\[ \mu_{f_i} = f_c \]  

(7)

\[ \sigma_{f_i}^2 = \frac{B^2}{24\gamma} \]  

(8)

In MFSK systems, the frequency of the carrier is allowed to take one of \( M \) possible values, the transmitted waveform corresponding to any one of the \( M \) symbols is given by

\[ s_k(t) = A \cos(2\pi f_k t) = A \cos(2\pi(f_c + if_d)t) \]  

(9)

One may note that the noisy MFSK signal can be treated as a signal formed by interleaving the random vectors \( S_k(t) \) \( i = 1,2, ..., M - 1 \) of individual symbols. Expressions for the mean and variance of such a random vector are derived from the mean and variance of the component random vectors in Appendix A. From the results of the Appendix A, we get

\[ \mu_{f_i(M)} = f_c \]  

(12)

\[ \sigma_{f_i}^2(M) = \frac{1}{M} \sum_{k=1}^{M} \sigma_{\mu k}^2(M) + \frac{1}{M} \sum_{k=1}^{M} \mu_k^2(M) \]  

(13)

\[ \sigma_{\mu k}^2(M) - \frac{1}{M^2} \sum_{k=1}^{M} \sum_{k=1}^{M} \mu_k(M) \mu_k(M) \]
For a given \( f_s \) and \( \gamma \) eq. (13) is a monotonic increasing function of M, the number of frequency states.

In M-ary PSK systems, the phase of carrier is allowed to take one of M possible values. However, the instantaneous frequency of this waveform is constant except at the phase transitions where it is theoretically infinite. If the signal is discrete in time, the instantaneous frequency at the phase transitions can not take infinite value but it does attain a very large value. In the presence of additive gaussian noise the instantaneous linear frequency assumes gaussian density with a mean and variance given by eq.(7) and eq.(8) respectively.

In the section to follow, we derive a decision statistic based on variance of instantaneous frequency MPSK and MFSK signals.

### III. Discrimination Between MPSK and MFSK Signals

Consider the eq. (12) which gives the variance of instantaneous frequency \( \sigma_f^2(M) \) of a noisy MFSK signal. From this equation, we note that the variance is a function of an amplitude \( \gamma \), frequency deviation \( f_d \) and the number of frequency states \( M \). As explained earlier, the variance \( \sigma_f^2(M) \) is a monotonic function of \( M \) and hence \( \sigma_f^2(M) < \sigma_f^2(M) \) for \( M > 1 \).

From the eq.(6), we note that the variance of instantaneous frequency of MPSK signals is a function of amplitude \( \gamma \). To illustrate the nature of dependence of variances of \( f_s \) for both MPSK and MFSK, a plot of \( \sigma_f^2(M) \) as a function of \( \gamma \) for a bit rate \( r_b = 250 \text{bps} \) and a frequency deviation \( f_d = 500 \text{Hz} \) is shown in Fig 1a. From this plot we note that the variances \( \sigma_f^2(M) \) are quite different for MPSK and MFSK signals and it is possible to discriminate these two types of modulations. However, since these variances change as a function of CNR and in a real world scenario we have no apriori knowledge of the available CNR, it is preferable that the decision statistic be independent of CNR. In what follows a technique to eliminate this dependence is presented.

Consider a pair of concentric band pass filters having bandwidths \( B_1 \) and \( B_2 \) (\( B_2 = K \cdot B_1 \); \( K \) being a real number greater than unity). Let the received signal be passed through these filters. The variance of instantaneous frequency of a MPSK signal at the output of these filters can be obtained from eq.(8) as

\[
(\sigma_f^2)_{psk}^{1} = \frac{B_1^2}{24\gamma_1}; \quad (\sigma_f^2)_{psk}^{2} = \frac{B_2^2}{6\gamma_2}
\]

where \( \gamma_1 \) and \( \gamma_2 \) are the CNRs at the output of the filters of bandwidths \( B_1 \) and \( B_2 \) respectively. We note that \( \gamma_1 = \gamma_2/K \), if \( B_2 = KB_1 \). Thus the variance expressions reduce to

\[
(\sigma_f^2)_{psk}^{1} = \frac{B_1^2}{24\gamma_1}; \quad (\sigma_f^2)_{psk}^{2} = \frac{K^3B_1^2}{24\gamma_1}
\]

The above equation suggests that by forming a ratio of variances, we get a parameter that becomes independent of \( \gamma \) for MPSK signal, which is given by

\[
R_{PSK} = \frac{(\sigma_f^2)_{psk}^{1}}{(\sigma_f^2)_{psk}^{2}} = K^3
\]

An additional advantage is that the ratio can be set by choosing an appropriate value of \( K \).

Now, let us consider the ratio of variances for the case of MFSK signals. Using eq.(13) for the \( \sigma_f^2(M) \) of FSK at two different bandwidths and after some algebraic manipulation, we obtain the ratio as

\[
R_{FSK} = \frac{K^3 + \beta}{\beta + 1}
\]

where

\[
\beta = 2f_d^2 \frac{(M^2 - 1)\gamma_1}{B_1^2}
\]

which obviously has a dependence on CNR(\( \gamma \)) and the frequency deviation \( f_d \). The bandwidth \( B_f \) must be selected in such a way that signal components of the received signal are not lost or highly attenuated and at the same time keeping the noise entering the filter as low as possible. One of the ways for choosing appropriate \( B_f \) is to identify the significant portion of the spectrum by any one of the spectral estimation methods and choose \( B_f \) so as to pass the significant portions. From the power spectra of MFSK signals \[13\] it is easy to arrive at the required bandwidth of the filter \( B_f \) as

\[
B_f = (M + 2)f_d
\]

Substituting eq.(19) in eq.(18) we get

\[
R_{FSK} = \frac{2(M^2 - 1)\gamma_1}{(M + 2)^2}
\]

Thus, for appropriately chosen filter bandwidth, \( R_{FSK} \) is independent of \( f_d \). To illustrate the behavior of \( R_{FSK} \), both \( R_{PSK} \) and \( R_{FSK} \) are plotted as a function of \( \gamma \) and \( M \), for \( K = 1.5 \) and \( f_d = 500 \text{Hz} \) in Fig 1b. From these plots we note that one can set a threshold of \( T_{th} \) (here it is 3)
on the decision static to discriminate between MPSK and MFSK signals up to a CNR as low as 0dB.

**IV. Proposed Novel Classifier**

Though the analytical expressions for the variance Ration $R$ are very promising for classification of digital angle modulated signals, some practical issues hamper the effectiveness of the classifier at the low CNRs. As mentioned earlier, the instantaneous frequency of MPSK signal will have spikes at phase transitions. The variance expressions derived earlier do not take this in to account. Thus, while implementing the proposed method, these spikes have to be suppressed. For the elimination of spikes in the instantaneous frequency, an impulse elimination filter which does not effect the flat portion of the instantaneous frequency is to be used. One of the simplest methods used for such purposes is the median filter. A median filter [14] of a size $N_w$ samples can suppress all the spikes having width of less than or equal to $N_w$.

Thus the proposed algorithm for discriminating MPSK and MFSK modulated signals can be stated as follows.

**Step 1:** Compute the averaged short time fourier transform (STFT) of the given noisy modulated signal $s(t)$.

**Step 2 :** Using threshold and peak detection algorithm identify the spectral band of signal activity.

**Step 3 :** Estimate the approximate centroid and bandwidth of the above identified signal band. Call them as $\bar{f}_c$ and $\bar{B}_1$.

**Step 4 :** Design a band pass filter centered at $\bar{f}_c$ and having a bandwidth $\bar{B}_1$.

**Step 5 :** Design a second band pass filter centered at $\bar{f}_c$ and having a bandwidth $\bar{B}_2$.

**Step 6 :** Pass the noisy modulated signal $s(t)$ through the two band pass filters separately. Let the output signals be $y_1(t)$ and $y_2(t)$ respectively.

**Step 7 :** Compute the analytical envelopes of the outputs of both the band pass filters.

**Step 8 :** Compute the instantaneous frequencies of both analytical envelopes.

**Step 9 :** Remove the spikes in the instantaneous frequencies using a median filter of size $N_w$.

**Step 10 :** Estimate the variances $(\hat{\sigma}_i^2)_1$ and $(\hat{\sigma}_i^2)_2$ of the median filtered instantaneous frequencies.

**Step 11 :** Compute the variance ratio $(\hat{\sigma}_i^2)_2/(\hat{\sigma}_i^2)_1$.

**Step 12 :** Classify the signal as MPSK signal, if the variance ratio is greater than a threshold $T_{th}$, else classify it as MFSK signal.

**V. Simulation and Results**

To ascertain the theoretical predictions of the earlier sections, extensive computer simulations for discriminating between MPSK and MFSK signals were carried out. In these simulations, a random symbol sequence of length 128 with equal probability is taken as the message data. First $M \times 128$ random bits are generated and then each set of $M$ bits is converted into a symbol, thus making a total of 128 symbols. Thus for $M=2$, each bit is taken a symbol, while for $M=3$, each set of 3 bits makes a symbol. A bit rate ($r_b$) of 250 bps, a carrier frequency ($f_c$) of 4000Hz are used for generating MPSK or MFSK signals. A frequency deviation ($f_d$) of 500Hz is used in case of the MFSK signal. A sampling frequency ($f_s$) of 22 KHz is used in all simulations.

The MPSK signals were generated using

$$s(t) = A \cos(\omega t + \varphi_0 + \varphi_k)$$  \hspace{1cm} (21)

where $\varphi_k = \frac{2ni}{M}$; $i = 0, 1, ..., M - 1$ and the continuous phase MFSK signals were obtained from
\[ s(t) = A \cos \left( 2\pi \left( f_c + (2k - 1 - M) \frac{f_s}{2} \right) t + \varphi_0 \right) \] 

The initial phase \( \varphi_0 \) is set to be zero for simplicity. A zero mean white gaussian noise (WGN) of variance \( \sigma_w^2 \) was added to form the noisy versions of these signals. The variance \( \sigma_w^2 \) of the noise is chosen so as to give the required Carrier-to-Noise Ratio (\( \gamma \)). Simulations are carried out for \( \gamma \) of 30dB, 20dB, 10dB, 5dB, 3dB, 2dB and 0dB.

The proposed novel classifying algorithm is applied on the noisy signals. First the signal activity band of the noisy signal is identified by the STFT analysis done by a customized matlab function \textit{mySTFTanalysis()}. The band width of the signal is maximum for 8FSK signal and is found to be 2250Hz approximately which is nine times the bit rate, for the frequency deviation of 500Hz. For PSK signal bandwidth is almost constant and is around 500Hz, which is close to the theoretical value of \( 2f_b \). The estimated spectral centroid \( \tilde{f}_c \) is around 4000Hz, close to theoretical value of \( f_c \). The spectral centroid estimation is done by an algorithm as described in [16]. The bandwidth including the first side lobe is found to be equal to the symbol rate i.e. 250Hz, 125Hz and 166Hz corresponding to \( M=2,3 \) and 4 respectively.

The noisy signal is filtered using a pair of concentric band pass filters centered on \( \tilde{f}_c \) and having bandwidths \( B_1 \). These filters are implemented as 6-th order type I chebyshev filters with a pass band ripple (\( \delta \)) of 40dB or maximally flat butterworth filters. The ratio of bandwidths \( K \) is set at 1.5.

An analytical envelope \( s(t) + j \hat{s}(t) \) computed for the output of each band pass filter. The hilbert transform \( \hat{s}(t) \) is obtained by using a customized matlab function \textit{myHilbert()}. Then, instantaneous frequency is estimated using the eq.(2) and median filtered to eliminate the spikes at the phase transitions that occur at the symbol boundaries. The size of the median filter is set as 5 samples. The variances of the median filtered instantaneous frequencies \( \left( \tilde{\sigma}_f^2 \right)_1 \) and \( \left( \tilde{\sigma}_f^2 \right)_2 \) were computed as the unbiased sample variances. From these the estimate of the variance ratio \( \tilde{R} \) is obtained.

The noisy PSK and FSK signals are simulated as several realizations of random message bits and an additive gaussian noise. Here 128 random realizations of each modulated signals are carried out. Thus for \( M=2,3,4 \) (3 cases), for \( \text{CNR}=30 \text{dB}, 20 \text{dB}, 10 \text{dB}, 5 \text{dB}, 3 \text{dB} \) and 2dB (6 cases), for PSK and FSK modulations (2 cases) and for 128 realizations (128 cases), a total of \( 3 \times 6 \times 2 \times 128 = 4608 \) noisy modulated signals are generated.

In what follows, the results of the simulations are presented in detail from Fig.2 through Fig 11. Figs.2 and Fig.3 give the averaged short time fourier transform (STFT) of noisy MPSK and MFSK signals respectively. From these spectrum plots, the band of signal activity can be determined in each case. Here, the band containing the main lobe and one side lobe on either side of main lobe is considered as the bandwidth of the MPSK/MFSK signals. Thus from Fig 2a, the bandwidth of 2PSK is found to be 1000Hz. This value is determined automatically by peak detection and threshold process applied on the spectrum. Similarly it is found to be 500Hz and 330Hz for 4PSK and 8PSK signals respectively from Fig 2b and 2c. Similar the bandwidths for MFSK signals are estimated to be 2000Hz, Fig 3. These values are set to be the band pass filter and signal is processed. At the output of the filter the analytical envelope and the instantaneous frequency is found in each case. The instantaneous frequency is passed through a median filter of size 15. Figs 4, 5 and 6 give the instantaneous frequency at the band pass filter output, its histogram, instantaneous frequency at the median filter output and its histogram for 2PSK, 4PSK and 8PSK signals respectively. It may be observed that the zero values frequently appearing in the instantaneous frequency are actually the artifacts. The spike at the 0 in the histogram of the instantaneous frequency corresponds to these artifacts. In computations and in plotting, these artifacts are avoided by replacing them with NaN. Matlab does not show these points in the instantaneous frequency plots and hence the gaps in the plots. The instantaneous frequency and corresponding histograms for MFSK signals are shown in Figs 7, 8 and 9 for \( M=2,4 \) and 8 respectively. Please note that in these figures each frequency state appears as a Gaussian shaped hump in the composite histogram. The variance or spread of these Gaussian shaped humps reduces after median filtering clearly bringing out the M frequency states in the histogram even at low CNRs.
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**Figure 2**: The averaged short time fourier transform (STFT) of noisy (a) 2PSK signal (b) 4PSK signal (c) 8PSK signal (CNR=30dB)

**Figure 3**: The averaged short time fourier transform (STFT) of noisy (a) 2FSK signal (b) 4FSK signal (c) 8FSK signal (CNR=30dB)

**Figure 4**: (a) The Instantaneous Frequency of noisy 2PSK signal $s(t)$ (b) Histogram of Instantaneous Frequency (c) The Instantaneous Frequency of First Band Pass Filter ($B_1=1000$ Hz) Output after median filtering (d) Histogram of median filtered Instantaneous Frequency (CNR=3dB)
Fig. 45. (a). The Instantaneous Frequency of noisy 4PSK signal $s(t)$ (b). Histogram of Instantaneous Frequency (c). The Instantaneous Frequency of First Band Pass Filter ($B_1=500\text{Hz}$) Output after median filtering (d). Histogram of median filtered Instantaneous Frequency (CNR=3dB)

Figure 6: (a). The Instantaneous Frequency of noisy 8PSK signal $s(t)$ (b). Histogram of Instantaneous Frequency (c). The Instantaneous Frequency of First Band Pass Filter ($B_1=330\text{Hz}$) Output after median filtering (d). Histogram of median filtered Instantaneous Frequency (CNR=3dB)
Figure 7: (a). The Instantaneous Frequency of noisy 2FSK signal $s(t)$ (b). Histogram of Instantaneous Frequency (c). The Instantaneous Frequency of First Band Pass Filter Output after median filtering (d). Histogram of median filtered Instantaneous Frequency (CNR=20 dB)

Figure 8: (a). The Instantaneous Frequency of noisy 4FSK signal $s(t)$ (b). Histogram of Instantaneous Frequency (c). The Instantaneous Frequency of First Band Pass Filter Output after median filtering (d). Histogram of median filtered Instantaneous Frequency (CNR=20 dB)

Figure 9: (a). The Instantaneous Frequency of noisy 8FSK signal $s(t)$ (b). Histogram of Instantaneous Frequency (c). The Instantaneous Frequency of First Band Pass Filter Output after median filtering (d). Histogram of median filtered Instantaneous Frequency (CNR=20 dB)
Figure 10: (a). Variance ratio (R) of Instantaneous Frequencies at the output of two filters for different CNRs- top plot for 30dB, middle plot for 20dB and bottom plot for 10dB.

Figure 11: Variance ratio (R) of Instantaneous Frequencies at the output of two filters for different CNRs- top plot for 5dB, middle plot for 3dB and bottom plot for 0dB.

Finally the variance ratios for different cases of PSK and FSK signals are presented in Figs 10 and 11 respectively. In each figure the variance ratio vector is plotted for 128 random realizations of the noisy signal. In Fig 10, the variance ratio for 2PSK signal (cyan curve) wanders around 3 which is very close to the theoretically derived value $K^2 = 1.5^2 = 3.375$. For 4PSK and 8PSK signals his value is 1.5 in stead of 3. The value of 3 can be obtained in these cases too by properly selecting the bandwidth $B_j$. However, this is not going to be a limitation for our FSK/PSK signal classification problem, since a threshold $T_m$ of 1.4 would serve the purpose even at 0dB. With this threshold value Monte Carlo simulations were carried out for these signals and the misclassification error is found to be minimal as shown in table 1. The impressive performance of the proposed technique even at such a low CNR is attributed to the implicit CNR improvement offered by the band pass filters.

Table 1: Misclassification Error (%)

<table>
<thead>
<tr>
<th>CNR/Modulation</th>
<th>30dB</th>
<th>20dB</th>
<th>5dB</th>
<th>3dB</th>
<th>0dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>2/4/8PSK</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>2/2/8FSK</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The discrepancies in the variances and the variance ratios observed between the theoretical value $R$ and estimated value $\hat{R}$ in all cases, which are attributed to the following:

1. The expression for the variance of the instantaneous frequency assumes an ideal band pass filter whose response is different from that of the 6-th order chebyshev or butterworth filters used in the simulations. (No notable difference is observed in the results by changing the filter type: chebyshev or butterworth. This is expected because we are finding the variance ratios and the trend in variance is the same in both numerator and denominator.)
2. The effect of spikes in the instantaneous frequency that occur at the symbol boundaries are not considered in the theoretical derivations.

3. The analytical derivations does not include the effect of median filter used in the simulations to eliminate the spikes in the instantaneous frequency.

One more important point is that the values of $t_0$, $t_e$ and $t_a$ used in the simulations are on lower side and closer to the values found in some ITU V series data modems [15]. These values are considered only for reducing the computational requirements. This, however, is not a limitation of the theory and the proposed algorithm developed in earlier sections.

VI. CONCLUSIONS AND FUTURE WORK

In this paper a novel classification scheme based on the variance of instantaneous frequency to discriminate between noisy M-ary Phase Shift keyed (MPSK) and M-ary Frequency shift keyed (MFSK) signals is proposed. In the proposed method, the received signal is passed through the pair of band pass filters and the ratio of variances of instantaneous frequency of the filter outputs is used as decision statistic. Analytical expressions are developed for the decision statistic. These expressions show that the discrimination between PSK and FSK is possible even at a carrier-to-noise ratio (CNR) of 0dB. The satisfactory performance of the proposed technique even at such a low CNR is attributed to the implicit CNR improvement at the output of the band pass filters. Simulation results validate the theoretical predictions made and the analytical expressions derived. The effect of changing the filter bandwidths and the median filter size on the decision static and the classification within PSK or FSK group ($M=2$ or 4 or 8) can be considered as the extension of this work.

VII. APPENDIX A

Consider a set of independent random vectors $\{X_i, i= 1,2, \ldots , M\}$ with respective means $\{\mu_i\}$ and standard deviations $\{\sigma_i\}$. Let $Y$ be a random vector formed from $\{X_i\}$ such that

$$Y = [X_1, X_2, \ldots , X_M]$$

(A.1)

The mean of $Y$ is

$$\mu_Y = E\{Y\} = \frac{1}{M} [E\{X_1\} + E\{X_2\} + \ldots + E\{X_M\}]$$

and

$$\mu_Y = \frac{1}{M} [\mu_1 + \mu_2 + \ldots + \mu_M] = \frac{1}{M} \sum_{i=1}^{M} \mu_i$$

(A.2)

The Variance of $Y$ is

$$\sigma_Y^2 = E \left\{ (Y - \mu_Y)^2 \right\}$$

$$= E \left\{ \left[ (X_1 - \mu_y)^2 + (X_2 - \mu_y)^2 + \ldots + (X_M - \mu_y)^2 \right] \right\}$$

Expressing in terms of expectations of individual elements of the vector and substituting for $\mu$, we obtain

$$\sigma_Y^2 = \frac{1}{M} \sum_{i=1}^{M} E \left\{ (X_i - \frac{1}{M} \sum_{i=1}^{M} \mu_i)^2 \right\}$$

(A.3)

Let us add and subtract $\mu_i$ to the term in the bracket to get

$$\sigma_Y^2 = \frac{1}{M} \sum_{i=1}^{M} E \left\{ (X_i - \mu_i + \mu_i - \frac{1}{M} \sum_{i=1}^{M} \mu_i)^2 \right\}$$

(A.4)

By expanding the square term and using the properties of the Expectation operator, we obtain the variance expression as

$$\sigma_Y^2 = \frac{1}{M} \sum_{i=1}^{M} \sigma_i^2 + \frac{1}{M} \sum_{i=1}^{M} \mu_i^2 - \frac{1}{M} \sum_{i=1}^{M} \sum_{j=1}^{M} \mu_i \mu_j$$

(A.5)

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