

Mathematical Research in Digital Age

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Abstract

The time when someone can do real, publishable mathematics completely without the aid of a computer is coming to a close; the use of computers in mathematical research is both widespread and under-acknowledged. Mathematicians use computers in a number of ways. This paper highlights the importance of mathematics and digital age in today's technological advancement; it also explains the influence of digital age on Mathematics research. Key areas where Information and Communication Technology can be applied to Mathematical research are discussed. To demonstrate the use of computer program on Mathematical analysis, some problems were solved analytically and were also solved using computer programs (Mathlab and Python). These two procedures are compared and it is clearly shown that using computer packages to solve Mathematical problems are more efficient, easier and accurate.

Index terms— mathematical research, digital age, automated theorem proven, scientific computing

1 Introduction

Mathematics is often defined as the study of quantity, magnitude, and relations of numbers or symbols. It embraces the subjects of arithmetic, geometry, algebra, calculus, probability, statistics, and many other special areas of research. It is the study of structures and pattern in large numerical sets [1]. Mathematics is an indispensable subject of study; it plays an important role in forming the basis of all other sciences which deal with the material substance of space and time. It is said that Mathematics is the gate and key of the Science.

According to [2] Mathematics is a way of thinking, a way of organizing a logical proof, a way reasoning that gives an insight into the power of human mind. It is uniquely well placed to respond to the demand of rapidly changing fields of life such as engineering, biological sciences, medicine and economic.

Research in general is a way of investigating a system, model, matter or theorem to discover hidden or previously unknown fact.

2 a) Research in Mathematics

In mathematics, research calls for the creation of new results, that is, either new theorems radically different or improved proofs of older results. Research comprises of creative work undertaken on a systematic basis in order to increase the stock of knowledge, including knowledge of humans, culture and society, and the use of this stock of knowledge to devise new applications. It is used to establish or confirm facts, reaffirm the results of previous work, solve new or existing problems, support theorems, or develop new theories [3]. A research project may also be an expansion on past work in the field. To test the validity of instruments, procedures, or experiments, research may replicate elements of prior projects, or the project as a whole. According to [4] research is the systematic investigative process employed to increase or revise current knowledge by discovering new facts. In Mathematics research theorem can be proved as well as taking a number of pieces and constructing a worthwhile example by putting them together in a new way [3].

3 b) Digital age

The digital age also known as computer age, information age, new media age; is a period in human history characterized by the shift from traditional industry that the industrial revolution brought through industrialization, to an economy based on information computerization. The digital age is the time period starting in the 1970s with the introduction of the personal computer with subsequent technology introduced providing the ability to make work easier and faster. The digital age formed by capitalizing on computer microminiaturization advances the evolution of technology in daily life, as well as educational life style. Digital age has allowed rapid global communications and networking to shape modern society which we call Information and Communication Technology (ICT) world [5].

ICT is an umbrella term that includes any communication device or application, encompassing: radio, telephone lines and wireless signals, computers as well as necessary enterprise software, hardware, storage, and audio-visual systems, satellite systems which enable users to access, store, transmit, and manipulate information. It is also encompasses various services and applications associated with them, such as videoconferencing and distance learning. It stresses the role of unified communication and the integration of telecommunications [5].

4 c) Benefits of Digital age to Research

The digital age (Information Age) has affected the workforce in several ways.

5 Key Roles Digital Age has Played in Mathematical Research

Before the digital age, professional mathematicians did most of their work on desks using paper and pencil. Today mathematicians still sit at a desk facing monitor screens or laptops. The paper and pencil are still there but a lot of mathematician's activities now involve the use of computer. The computer does not simply assist mathematicians in doing business as usual; rather it changes the nature of what is done. Computers then have changed the way Mathematics progresses. There are many specific forms in which digital age has contributed to mathematical research, some of these forms are in problem solving task, exploring pattern and relationships, practicing of number skills, calculators, spreadsheets, databases and online, interactive resources, automated theorem proving, symbolic computation, scientific computing. a) Problem solving task Problem solving task in mathematics is about solving mathematical problems. The major aim of mathematics education is to equip researchers to solve problems. Mathematics consists of skills and processes. The skills are things that researchers are familiar with. These include the basic arithmetical processes and the algorithms that go with them. They also include algebra in all its levels as well as sophisticated areas such as the calculus. Problem solving task is a mathematical process. It is the side of mathematics that enables us to use acquired skills in a wide variety of situations. Now we shall consider some problems, by solving them manually and also using a computer program.

6 Example 1:

Given an initial value problem $y'' = 2y$, $y(0) = 1$, $y'(0) = 1$ (1)

Note [equation 1 is a linear second -order homogeneous differential equation]

7 Manual Solution

To solve this let assume that $y = e^{mx}$ is a solution to equation 1 Hence; finding the derivative we have, $y' = me^{mx}$ Differentiating further we have, $y'' = m^2 e^{mx}$ Putting these result back into equation 1, $m^2 e^{mx} = 2e^{mx}$ $m^2 = 2$ $m = \pm\sqrt{2}$

Factorizing, $m^2 - 2 = (m - \sqrt{2})(m + \sqrt{2}) = 0$ Divide both side by $(m - \sqrt{2})(m + \sqrt{2})$ we have, $m^2 - 2 = 0$ $(m - \sqrt{2})(m + \sqrt{2}) = 0$ $m = \sqrt{2}$ or $m = -\sqrt{2}$

Since the root of the characteristic equation is complex, hence the general solution of equation 1 is: $y(x) = e^{\sqrt{2}x} \cos x + e^{-\sqrt{2}x} \sin x$ (2)

Now applying the initial conditions $y(0) = 1$, $y'(0) = 1$ (1)

Let the basis solution to the special second order initial value problems (1) be the exponential function $y = e^{mx}$ $y' = me^{mx}$ $y'' = m^2 e^{mx}$ (2)

where m is a constant.

Expanding equation (2) $y = e^{mx}$ $y' = me^{mx}$ $y'' = m^2 e^{mx}$ (3)

Taking r in equation (3) to be the sum of number of interpolation points (I) and number of collocation points (C), $I = 2$ and $C = 3$, the approximate solution to equation (1) is: $y = 2/3 e^{\sqrt{2}x} + 1/3 e^{-\sqrt{2}x} + 1/4 e^{0x} + 1/4 e^{1x} + 1/3 e^{2x} + 1/2 e^{3x}$ (4)

Differentiating equation (3) twice gives;

... $6e^{mx} = m^2 e^{mx}$ (6)

Interpolating equation (3) at x_n and x_{n+1} $y = e^{mx}$ $y' = me^{mx}$ $y'' = m^2 e^{mx}$ (8)

Collocating equation (3) at x_n , x_{n+1} , x_{n+2} (7) and evaluating at x_{n+2} , to get the scheme below:

$$\begin{aligned} & \frac{1}{6} y''(x_{n+2}) - \frac{1}{2} y''(x_{n+1}) + \frac{1}{3} y''(x_n) = \frac{1}{6} m^2 e^{mx_{n+2}} - \frac{1}{2} m^2 e^{mx_{n+1}} + \frac{1}{3} m^2 e^{mx_n} \\ & = \frac{1}{6} m^2 e^{mx_{n+2}} - \frac{1}{2} m^2 e^{mx_{n+1}} + \frac{1}{3} m^2 e^{mx_n} \end{aligned}$$

99 h f h n n n n n n n n n n n n n n 2 2 2 1 1 2 2 2 2 4 2 3 2 2 1 + + + + + ? ? + + ? = ? [] h f f x h f h f x h
 100 f x f x h f h n n n n n n n n n n n n n n 2 2 2 1 1 2 3 2 4 4 4 2 3 2 1 + + + + + ? ? ? + ? = ? [] 2 1 2 4 2 1 +
 101 + + ? = n n n f f f h ? Putting the values of ? o , ? 1 , ? 2 , ? 3 , ? 4 into equation ([] n n n n n f f f h y y
 102 y + + + + ? + + + + 1 2 2 1 2 10 2 2 (12) Solution ?? ??+2 = 2?? ??+1 ? ?? ?? + ? 2 2 [?? ??+2 + 10??
 103 ??+1 + ?? ??](13)
 104 For n = 0 to 10 Solving for n = 0 The scheme becomes;?? 2 = 2?? 1 ? ?? 0 + ? 2 2 [?? 2 + 10?? 1 + ?? 0]
 105 Solving for y 1 since the scheme is an implicit scheme We use Taylor series as the predictor -corrector?? ?? =
 106 ????? ?? = ????? ?? = ????? ?? = ????? ?? ??+1 = ??(?? ?? + ?) = ??(?? ??) + ??? ? (?? ??) + 1
 107 2! ? 2 ?? ?? (?? ??) + 1 3! ? 3 ?? ?? (?? ??) + ?
 108 For n = 0, where h = 0.001 ?? 1 = 1 + (0.?? 2 = 2?? 1 ? ?? 0 + ? 2 2 [?? 2 + 10?? 1 + ?? 0] ?? 2 =
 109 2(1.000995) ? 1 + 0.001 2 2 [?1.001998 ? 10(1.000995) ? 1]
 110 Computing the values above, we have; ?? 2 = 1.001984
 111 Going back all over again to solve for n = 1?10 will take a lot of time and computational accuracy will not be
 112 there.
 113 Using A Python Program To Solve The Problem #program to calculate... Import math y = [0,0,0,0,0,0,0,0]
 114 f = [0,0,0,0,0,0,0,0,0,0] h = 0.001 #h = input("Enter the value for h: ") yp = 1 ypp = -1 yppp = -1 ypppp =
 115 1 yppppp = 1 ypppppp = -1 yppppppp = -1 y(0) = 1 y[0] = y0 + (h * yp) + (([i] = (2 * y[i-1]) -y0 + ((h**2)/2)
 116 * (f[i+1] + (10 * f[i]) + f[i-1]) n += 1 f[i] = f[i-1] f[i+1] = f[i] y0 = y[i-1]
 117 Automated Theorem Proving is an area of study to get computers to prove logical and mathematical
 118 statements. Not just enumerating instances of a theorem exhaustively, but applying logical deduction, induction,
 119 inference and search strategies (depth first, breadth first, best first, iterative deepening) to arrive at a solution.
 120 There are branches of Mathematics such as Model theory and Proof Theory which study proofs themselves.
 121 Automated theorem proving is a subfield of automated reasoning and mathematical logic dealing with proving
 122 mathematical theorem by computer programs. Example of Theorem-proving packages is Microsoft's Z3 [4].
 123 III.

8 Summary and Conclusion

125 Computers have changed the way mathematics progresses. The Digital age has made available for researchers
 126 indispensable hardware and software tools that can effortlessly assist in various ways to make research in
 127 mathematics easier, faster and motivating. This assistance covers areas such as number theory, calculus,
 128 differential equations and linear algebra among others. These resources come with many hundreds of built-
 129 in functions, extensive features for manipulating these functions, and a high-level computer language that allows
 130 one to easily create functions and procedures of their own. Mathematical research with the help of digital
 131 technology has made research to be easier and faster. Therefore mathematics research is more interesting and
 132 encouraging in digital age. ^{1 2}

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Figure 1: 2 ??H?Figure 1 :

Mathematical Research in Digital Age
Computing the values above, we have;

$$\delta_{??}''\delta_{??}'' 1 = ?1.000995$$

$$\delta_{??}''\delta_{??}'' ??+2 = 1 \ 2!$$

For $n = 0$, where $h = 0.001$

$$\delta_{??}''\delta_{??}'' 2 = ?1 + (0.001)(?1) + \text{Computing the values above, we have; } 1 \ 2 \quad (2??0.001) \ 2$$

$$\delta_{??}''\delta_{??}'' 2 = ?1.001998$$

$$\delta_{??}''\delta_{??}'' ?? = ??$$

Year For $n = 0$, where $h = 0.001$

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puter
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nology

$\delta_{??}''\delta_{??}'' 1 = ?1$ Putting the values back into the scheme below

$$\text{Computing the values above, we have; } 001)(1) \quad 1 \ 2 \quad (0.001) \ 2 \ (?1) +$$

+

$$?? \ 1 = 1.000995$$

$$\delta_{??}''\delta_{??}'' ??+1 = ???? (?? \ ?? + ?) = ???? (?? \ ??) + ??? \ ? \ ?? (?? \ ??) +$$

For $n = 0$, where $h = 0.001$

$$\delta_{??}''\delta_{??}'' 1 = ?1 + (0.001)(?1) +$$

1
2

Figure 2:

```

133     for j in range (10):
134         print(str(y[j]) + "\t") #Exact values exact = [0,0,0,0,0,0,0,0,0] print("-----[ EXACT VALUES
135 ]-----") x = h c = 0 if (x >= 0. ??-----[ DIFFERENCE ]-----") for e
136 in range (10) From the table above, the use of Python (a Programming Language) has enabled us to solve the
137 differential equation with different values of x. Furthermore, we are able to get the series of y-numerical and
138 y-exact with their difference without much stress.
139     With the help of digital age, the result obtained in the table above can be further interpreted by plotting a
140 graph that shows the difference between the ynumerical and y-exact.
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