

# Design of H $\infty$ Congestion Controller for TCP Networks Based on LMI Formulation

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## Abstract

In this paper, a state feedback H $\infty$  controller has been proposed in order to design an active queue management (AQM) system based on congestion control algorithm for networks supporting TCP protocols. In this approach, the available link bandwidth is modeled as a time-variant disturbance. The objective of this paper is to design controller which capable of achieving the queue size and guarantee asymptotic stability in the present of disturbance. An important feature of the proposed approach is that the performance of system, including the disturbance rejection and stability of closed-loop system, are guaranteed for all round-trip times that are less than a known value. The controller design is formulated in the form of some linear matrix inequalities, which can efficiently solved numerically. The simulation results demonstrate the effectiveness of proposed methods in comparison with other conventional methods.

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*Index terms*— TCP, AQM, time delay, H $\infty$ , LMI, stability, disturbance rejection.

## 1 INTRODUCTION

ommunication networks are an essential part of many applications in science and engineering, such as Web servers, multimedia, and remote control. However, traffic congestion is a major problem in today's Internet, because the quality of service cannot be guaranteed, since the number of users has grown rapidly and also unanticipated interference may occur. Therefore, congestion control techniques monitor network loads in an effort to anticipate and avoid congestion at common network bottlenecks. Congestion control is achieved through packet dropping.

Active queue management (AQM) [1,2] is a key congestion control scheme for reducing packet drops and improving network utilization. The random early detection (RED) [3] algorithm is the earliest well-known AQM scheme that eliminates the flow synchronization problem and attenuates the traffic load. Unfortunately, RED causes oscillations and instability due to the parameter variations. Therefore, some modified RED schemes, such as FRED [4] and SRED [5], have been proposed in the literature. However in those studies, both high network utilization and low packet loss cannot be guaranteed by only setting control parameters. Recently, control theory has been widely applied to the analysis and design of TCP networks and congestion control Author : Student, Lovely Professional University, Phagwara, Punjab, India. E-mail : aakanksha.artindia@gmail.com schemes for them. In [6], the theory of stochastic equations has been applied to develop a fluid-based model of the dynamics of the TCP and RED. Based on this TCP model, the fundamentals of control theory have been used to analyze and develop new AQM schemes. Proportional-integral (PI) controller was developed for a linearized system and implemented using differential equations [7]. In [8], a sliding mode variable structure control (SMVS) scheme for TCP congestion control has been developed. In [9], proportional-integral-derivative (PID) controllers have been proposed to improve the performance of TCP systems.

While great progress has been made in new congestion control schemes, some problems are still not sufficiently addressed. One important problem is the robustness of the congestion control algorithm against the disturbance on the available link bandwidth since it is often time-varying and cannot be exactly measured.

In this paper a  $\mathcal{H}$  state feedback control approach has been proposed. The main difference between our approach and the previous studies [10,11,12] is that, the approach proposed here uses a time-domain  $\mathcal{H}$  design method, which can deal with the situation where the round-trip time varies with time, while the previous studies use frequency-domain design method, which require that the system under consideration is time invariant.

Structure of the paper is as follows. In section II, system model and problem statement will be presented. We formulate our problem as proposed in [13]. In section III, we discuss how to deal with linear time delay systems. In the next section, a  $\mathcal{H}$  state feedback controller is employed to solve the problem for linear time varying systems. In section IV, performance of the closed loop system by using the proposed controller has been discussed in the form of some simulations and the paper is concluded in the section V.

## 2 II.

### 3 MODEL OF THE SYSTEM

We begin our discussion of AQM by introducing a dynamic model for TCP's congestion control.

In [6], a dynamic model of TCP behavior was developed by using fluid-flow and stochastic differential equation analysis. Similar to [13], here a simplified version of that model is used which neglects the TCP timeout mechanism. This model is described by the following equations:

$$\begin{aligned} \dot{W} &= \frac{1}{N} \left( \frac{C}{W} - p \right) W - \frac{1}{N} \frac{q}{W} \\ \dot{q} &= \frac{1}{N} \left( \frac{C}{W} - p \right) q - \frac{1}{N} \frac{q^2}{W} \end{aligned} \quad (1)$$

Where  $W$  is the TCP window size (in packets),  $q$  the queue length in the router (in packets),  $\tau$  the roundtrip time (in Sec),  $C$  the available link capacity (in packets/s),  $p$  T propagation delay (in Sec.),  $N$  the number of TCP sessions, and  $p$  the probability of packet mark. It is assumed that  $0 \leq q \leq C$  and  $0 \leq W \leq C$ ,

where  $q$  and  $W$  denote buffer capacity and maximum window size, respectively. The marking probability  $p$  belongs to the interval  $[0, 1]$ . In practical networks, the available link capacity changes with time and it is difficult to measure. Therefore it is taken as a disturbance in a lot of studies [14,15,16]. In this paper, it is supposed that the nominal value of  $C$ , say  $C_0$  is known, while  $\delta C$  is unknown, while  $\delta C$  is unknown, while  $\delta C$  is unknown.

$\delta C$  is unknown and considered as a disturbance for the system. Take  $(W, q)$  as the states and  $p$  as the input of the system. For a given triplet of network parameters  $(C_0, N, \tau)$ ,  $(C_0, N, \tau)$  any triplet  $(C_0, N, \tau)$  that is in the set  $\{(C_0, N, \tau) \mid C_0 \geq 0, N \geq 1, \tau \geq 0\}$  is a possible operating point. Now define  $\delta C = C - C_0$ ,  $\delta q = q - q_0$ ,  $\delta W = W - W_0$ .

We can obtain the linearized version of (1) as follows. When the delay appears only in the state variables, there are lots of results like [17,18]. But for the case where the time delay also appears in control variables, there is not any obvious solution. The linearized system is given by:

$$\begin{aligned} \dot{\delta W} &= \frac{1}{N} \left( \frac{C_0}{W_0} - p_0 \right) \delta W - \frac{1}{N} \frac{\delta q}{W_0} \\ \dot{\delta q} &= \frac{1}{N} \left( \frac{C_0}{W_0} - p_0 \right) \delta q - \frac{1}{N} \frac{\delta q^2}{W_0} \end{aligned} \quad (2)$$

The objective of this paper is to develop a  $\mathcal{H}$  design approach for the problem of AQM-based congestion control based on the dynamic model (2), which guarantees the ratio between the norms of some desired variables and that of the disturbance being less than some specified value. Furthermore, this specified value for the ratio can be minimized for a given group of network parameters. To this end, we will first study the  $\mathcal{H}$  control of general linear time delay systems and then apply the result to the above mentioned system.

### 4 III. $\mathcal{H}$ CONTROL OF LINEAR TIME DELAY SYSTEMS

As is well-known, the primary goal of a control algorithm is to guarantee that the closed-loop system is stable. For linear time-invariant single-input-single-output (SISO) plant without delay, this goal can be easily achieved by using classical controller design approaches, developed in 1950s and 1960s. Furthermore, the gain and phase margin indicated in these classical approaches provide a good measure for the robustness of closed-loop systems. However, it is difficult to apply these approaches to the controller design of a multi-input-multi-output (MIMO) plant or a time delay system. On the other hand, dealing with model uncertainty and disturbance is a main concern of control engineers. Therefore, various robust controller design approaches for complex plants have been developed since the 1980s. The  $\mathcal{H}$  design is one of those approaches.

A general setting for the  $\mathcal{H}$  design is illustrated in Fig. 1, where  $u$  is a control input,  $v$  the exogenous disturbance,  $z$  is the controlled output, and  $y$  is the measured output. The controlled output means the variable we want to regulate by designing a controller  $F$ . The objective of the  $\mathcal{H}$  control design is to find a controller  $F$  such that  $\|F(s)G(s)v(t)\|_2 < \gamma \|u(t)\|_2$ .

## 5 Global Journal of Computer Science and Technology

Volume XIII Issue IX Version I In contrast to the case of linear systems without delay, the solution for the  $\mathcal{H}$  control problem for time delay systems is quite different.  $\gamma$  describes a kind of disturbance rejection ratio between the controlled variable and the exogenous disturbance.

Clearly,  $\gamma$  describes a kind of disturbance rejection ratio between the controlled variable and the exogenous disturbance.



## 7 CONCLUSION

164 lower and upper barriers. This is realized in simulation by simply removing the constraint  $0 \leq k$  in the original  
165 model of the birth-death process and placing the new constraint  $(\cdot) \max \max k t k k \cdot \cdot \cdot$

166 on it. Note that in model (??), the delayed version of the exogenous disturbance also appears in the dynamics  
167 of the system. To take into account of this fact, define  $[ \cdot ] 1 0 = H$

168 for all cases to be studied. The approach proposed here is compared with the performance of P and PI  
169 controllers in [13]. Therefore, constant parameters of model extract from [13]. Where  $C=3750$  packets/s,  $N=60$   
170 flows and  $0 \leq =0.246$  Sec. For design of  $\cdot H$  controller based on LMI formulation, we first use constant parameters  
171 to calculate  $0 A$  ,  $1 A$  ,  $0 B$  ,  $1 B$  ,  $D$  matrices. Then we determine the maximum delay which the controller is  
172 robust against it. We solve (11) to find the state feedback  $K$  as follows:  $K = [ \cdot ] 005 - 2.3272e 003 - 1.5357e - [$

173 In all simulations, the initial windows size of every source and the initial queue length of the router are set to  
174 be zero. For each case controller, the same disturbance profile on the available link bandwidth is used for PI, P  
175 and  $\cdot H$  controllers. From Fig. 2, we can see the disturbance on the available link bandwidth which is the result  
176 of birth and death process with  $32 = \cdot a B$  and  $50 \max = k$  . From Fig. 3 and 4, we can see that, by using  
177 the  $\cdot H$  controller, a stable operating condition can be built up and maintained even in the situations where the  
178 available link bandwidth is subjected to presence of disturbance and the round-trip varies with different TCP  
179 sessions, while PI controller fail to do so. This is due to the lag of the response of the conventional PI controller  
180 to the sudden change of the network operating condition.

181 The responses of the queue size for the duration of time from 0 to 5 s can be observed in Fig. 3 and 4  
182 respectively. It is clear that both  $\cdot H$  and P controllers yield lower overshoot than PI, and yield almost bigger  
183 rise time than PI. Also from Table 1, it is obvious that the maximum overshoot in  $\cdot H$  is smaller than P and PI  
184 controllers in both queue length and window size states. V.

## 185 7 CONCLUSION

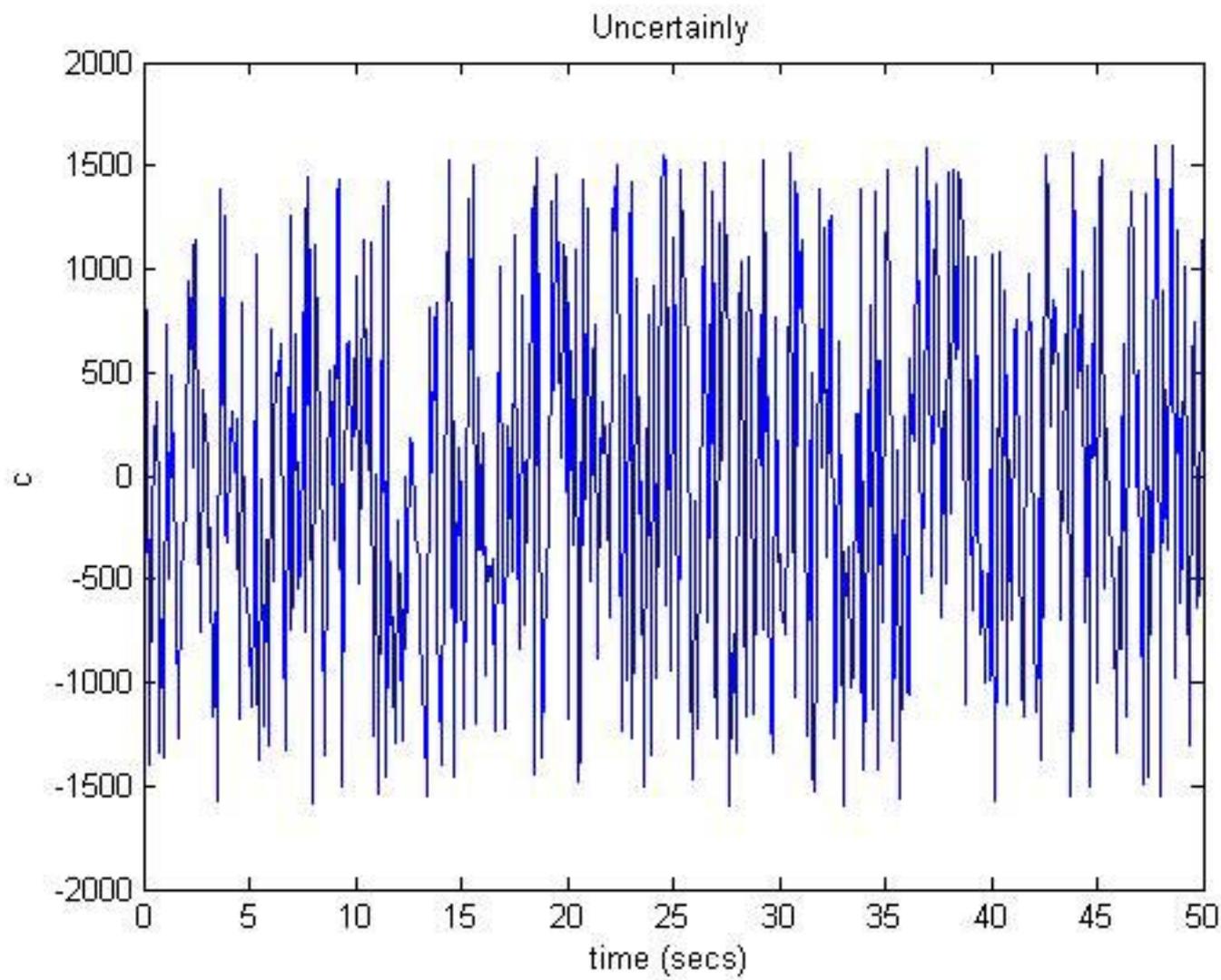
186 In this paper, a new design method for the  $\cdot H$  congestion controller of the TCP has been developed based on  
187 the LMI technique. In the approach, the available link bandwidth is modeled as a nominal constant value, which  
188 is known to the link, plus a timevariant disturbance, which is unknown.

189 The proposed approach can theoretically guarantee the system performance, including the disturbance rejection  
190 and the implied stability of the closed-loop system for all round-trip times that are less than a known value.  
191 Finally, it is pointed out that the effectiveness of the proposed approach has been verified only via simulation in  
192 Matlab. Further verification via packet-based simulation tools such as NS2 or via experimental studies is needed.

1 2 3

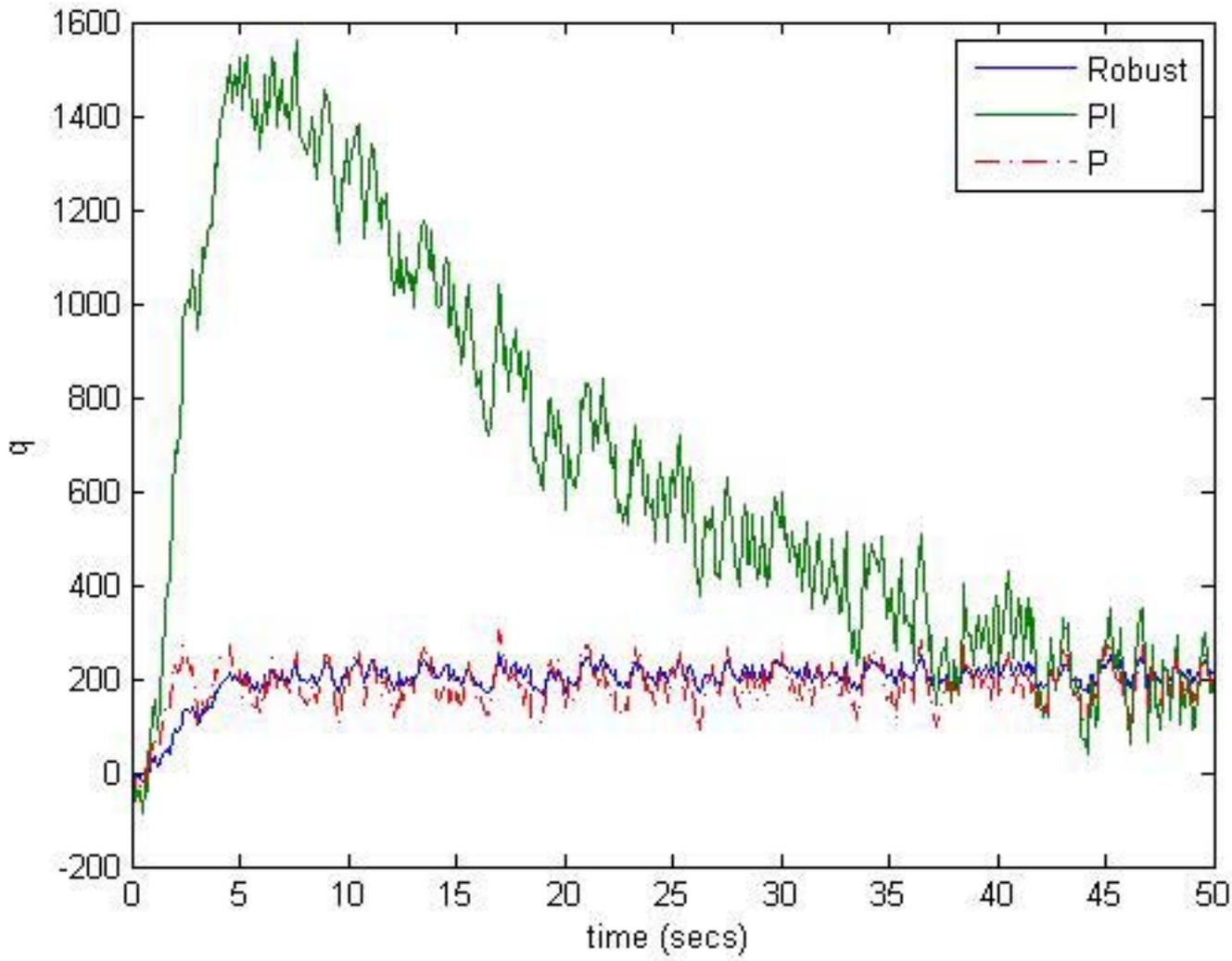


Figure 1: Figure 1 :



1

Figure 2: Lemma 1 :



3

Figure 3: 3 Q

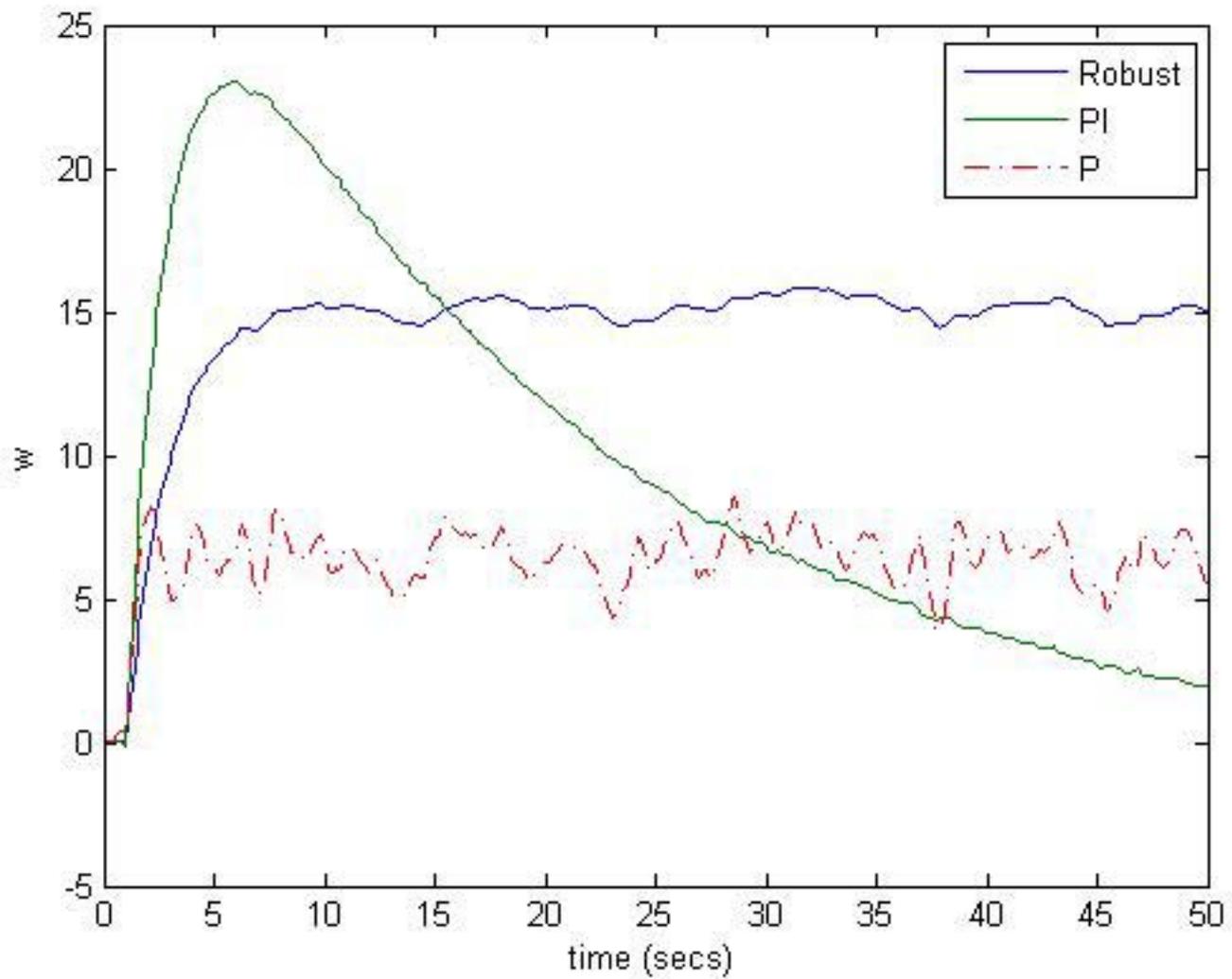


Figure 4:

1

controllers

Characteristics

Queue length maximum overshoot 6.6% 50%

Window size maximum overshoot 25% 67.5% 50%

H? PI P  
40%

Figure 5: Table 1 :

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<sup>1</sup>Following coupled and nonlinear delay-differential equations:

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<sup>3</sup>Design of H Congestion Controller for TCP Networks Based on LMI Formulation ?

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