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Set Theoretic Rajan Transform and its Properties

By G. Prashanthi, G. Sathya, Manish Prateek & E. G. Rajan

University of Petroleum and Energy Studies

Abstract- In this paper, we describe the formulation of a novel transform called Set Theoretic Rajan Transform (STRT) which is an extension of Rajan Transform (RT). RT is a coding morphism by which a number sequence (integer, rational, real, or complex) of length equal to any power of two is transformed into a highly correlated number sequence of same length. STRT was introduced by G. Sathya. In STRT, RT is applied to a sequence of sets instead of sequences of numbers. Here the union (\cup) is analogous to addition (+) operation and symmetric difference (\sim) is analogous to subtraction (-). This transform satisfies some interesting set theoretic properties like Cyclic Shift Invariance, Dyadic Shift invariance, Graphical Inverse Invariance. This paper explains in detail about STRT and all of its set theoretic properties.

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Set Theoretic Rajan Transform and its Properties

G. Prashanthi^a, G. Sathya^a, Manish Prateek^p & E. G. Rajan^ω

Abstract- In this paper, we describe the formulation of a novel transform called Set Theoretic Rajan Transform (STRT) which is an extension of Rajan Transform (RT). RT is a coding morphism by which a number sequence (integer, rational, real, or complex) of length equal to any power of two is transformed into a highly correlated number sequence of same length. STRT was introduced by G. Sathya. In STRT, RT is applied to a sequence of sets instead of sequences of numbers. Here the union (U) is analogous to addition (+) operation and symmetric difference (~) is analogous to subtraction (-). This transform satisfies some interesting set theoretic properties like Cyclic Shift Invariance, Dyadic Shift invariance, Graphical Inverse Invariance. This paper explains in detail about STRT and all of its set theoretic properties.

I. INTRODUCTION

In STRT, given a sequence of sets $X(n)$ of length N , which is a power of two, first it is divided into the first half and the second half each consisting of $(N/2)$ points so that the following holds good :

$$\begin{aligned}G(j) &= X(i)UX(i+(N/2)); \quad 0 \leq j \leq (N/2); \quad 0 \leq i \leq (N/2) \\H(j) &= X(i) \sim X(i-(N/2)); \quad 0 \leq j \leq (N/2); \quad 0 \leq i \leq (N/2)\end{aligned}$$

Now each $(N/2)$ -point segment is further divided into two half's each consisting of $(N/4)$ points so that the following holds good:

$$\begin{aligned}G1(k) &= G(j)UG(j+(N/4)); \quad 0 \leq k \leq (N/4); \quad 0 \leq j \leq (N/4) \\G2(k) &= G(j) \sim G(j-(N/4)); \quad 0 \leq k \leq (N/4); \quad 0 \leq j \leq (N/4) \\H1(k) &= H(j)UH(j+(N/4)); \quad 0 \leq k \leq (N/4); \quad 0 \leq j \leq (N/4) \\H2(k) &= H(j) \sim H(j-(N/4)); \quad 0 \leq k \leq (N/4); \quad 0 \leq j \leq (N/4)\end{aligned}$$

This process is continued till no more division is possible. The total number of stages thus turns out to be $\log_2 N$. Then the signal flow graph for STRT of length eight would be of the form shown in the Fig. 1.

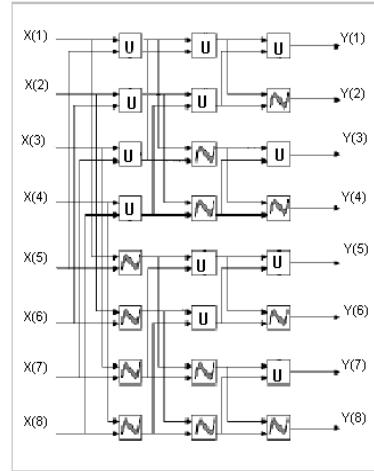


Fig. 1: Signal Flow graph of STRT

Unlike RT, duality doesn't hold good in STRT. If $X(n)$ is a set sequence of length $N=2k$, $k>0$ then its Set Theoretic Rajan Transform is denoted by $Y(k)$. Consider a set sequence $X(1)=\{1,2\}$, $X(2)=\{3,4,6\}$, $X(3)=\{4,5\}$, $X(4)=\{1,5\}$, $X(5)=\{1,4,5\}$, $X(6)=\{3,4,5\}$, $X(7)=\{2,6\}$, $X(8)=\{1,4,6\}$. Then STRT is computed as follows.

Input set sequence	Stage #1	Stage #2	Stage #3 STRT spectrum
{1,2}	{1,2,4,5}	{1,2,3,4,5,6}	{1,2,3,4,5,6}
{3,4,6}	{3,4,5,6}	{1,3,4,5,6}	{2,3}
{4,5}	{2,4,5,6}	{1,6}	{1,3,6}
{1,5}	{1,4,5,6}	{1,3}	{3,6}
{1,4,5}	{2,4,5}	{2,4,5,6}	{2,4,5,6}
{3,4,5}	{5,6}	{4,5,6}	{2}
{2,6}	{2,4,5,6}	{6}	{4,6}
{1,4,6}	{4,5,6}	{4}	{4,6}

II. ALGEBRAIC PROPERTIES OF STRT

STRT satisfies few interesting properties like Cyclic Shift Invariance, Graphical Inverse Invariance, Dyadic Shift invariance. All these properties are discussed below.

a) Cyclic Shift Invariance

Let us consider the same set of sequences $X(n)=\{1,2\}, \{3,4,6\}, \{4,5\}, \{1,5\}, \{1,4,5\}, \{3,4,5\}, \{2,6\}, \{1,4,6\}$. Using this set of sequences, one can generate seven more cyclic shifted versions such as $Xc1=\{1,4,6\}, \{1,2\}, \{3,4,6\}, \{4,5\}, \{1,5\}, \{1,4,5\}, \{3,4,5\}, \{2,6\}$; $Xc2=\{2,6\}, \{1,4,6\}, \{1,2\}, \{3,4,6\}, \{4,5\}, \{1,5\}, \{1,4,5\}, \{3,4,5\}$; $Xc3=\{3,4,5\}, \{2,6\}, \{1,4,6\}, \{1,2\}$,

Author a: Director, Computer Vision, Pentagram Research Centre Pvt. Ltd., Hyderabad, India. e-mail: prashanthi89@gmail.com

Author o: Director, Artificial Intelligence, Pentagram Research Centre Pvt. Ltd., Hyderabad, India. e-mail: sathyag413@gmail.com

Author p: Professor and Dean, University of Petroleum and Energy Studies, Dehradun Uttarakhand, India.
e-mail: mpreateek@ddn.upes.ac.in

Author ω: Adjunct Professor, University of Petroleum and Energy Studies, Dehradun Uttarakhand, India. e-mail: dr.rajaneg@gmail.com

$\{3,4,6\}, \{4,5\}\{1,5\}, \{1,4,5\}; Xc4=\{1,4,5\}, \{3,4,5\}, \{2,6\}, \{1,4,6\}, \{1,2\}, \{3,4,6\}, \{4,5\}\{1,5\}; Xc5=\{1,5\}, \{1,4,5\}, \{3,4,5\}, \{2,6\}, \{1,4,6\}, \{1,2\}, \{3,4,6\}, \{4,5\}; Xc6=\{4,5\}, \{1,5\}, \{1,4,5\}, \{3,4,5\}, \{2,6\}, \{1,4,6\}, \{1,2\}, \{3,4,6\}; Xc7=\{3,4,6\}, \{4,5\}, \{1,5\}, \{1,4,5\}, \{3,4,5\}, \{2,6\}, \{1,4,6\}, \{1,2\}.$ It is obvious that the cyclic shifted version of $Xc7(n)$ is $X(n)$ itself. One can easily verify that all these eight sequences have the same $Y(k)$, that is $\{1,2,3,4,5,6\}, \{2,3\}, \{1,3,6\}, \{3,6\}, \{2,4,5,6\}, \{2\}, \{4,6\}, \{4,6\}$.

b) Graphical Inverse Invariance Property

Consider a sample sequence $X(n) = \{1,2\}, \{3,4,6\}, \{4,5\}\{1,5\}, \{1,4,5\}, \{3,4,5\}, \{2,6\}, \{1,4,6\}$ and its has Graphical Inverse $X-1(n)=\{1,4,6\}, \{2,6\}, \{3,4,5\}, \{1,4,5\}, \{1,5\}, \{4,5\}, \{3,4,6\}, \{1,2\}$. Using this sequence one can generate seven more cyclic shifted versions such as $Xc1-1(n)=\{1,2\}, \{1,4,6\}, \{2,6\}, \{3,4,5\}, \{1,4,5\}, \{1,5\}, \{4,5\}, \{3,4,6\}; Xc2-1(n)=\{3,4,6\}, \{1,2\}, \{1,4,6\}, \{2,6\}, \{3,4,5\}, \{1,4,5\}, \{1,5\}, \{4,5\}; Xc3-1(n)=\{4,5\}, \{3,4,6\}, \{1,2\}, \{1,4,6\}, \{2,6\}, \{3,4,5\}, \{1,4,5\}, \{1,5\}; Xc4-1(n)=\{1,5\}, \{4,5\}, \{3,4,6\}, \{1,2\}, \{1,4,6\}, \{2,6\}, \{3,4,5\}, \{1,4,5\}; Xc5-1(n)=\{1,4,5\}, \{1,5\}, \{4,5\}, \{3,4,6\}, \{1,2\}, \{1,4,6\}, \{2,6\}, \{3,4,5\}; Xc6-1(n)=\{3,4,5\}, \{1,4,5\}, \{1,5\}, \{4,5\}, \{3,4,6\}, \{1,2\}, \{1,4,6\}, \{2,6\}; Xc7-1(n)=\{2,6\}, \{3,4,5\}, \{1,4,5\}, \{1,5\}, \{4,5\}, \{3,4,6\}, \{1,2\}, \{1,4,6\}$. It is obvious that the cyclic shifted version of $Xc8-1(n)$ is $X-1(n)$ itself. One can easily verify that all these eight sequences have the same $Y(k)$, that is, $\{1,2,3,4,5,6\}, \{2,3\}, \{1,3,6\}, \{3,6\}, \{2,4,5,6\}, \{2\}, \{4,6\}, \{4,6\}$.

c) Dyadic Shift Invariance property

The term 'dyad' refers to a group of two, and the term 'dyadic shift' to the operation of transposition of two blocks of elements in a sequence. For instance, let us take $X(n)=\{1,2\}, \{3,4,6\}, \{4,5\}\{1,5\}, \{1,4,5\}, \{3,4,5\}, \{2,6\}, \{1,4,6\}$ and transpose its first half with the second half. The resulting sequence $Td(2)[Xn]=\{1,4,5\}, \{3,4,5\}, \{2,6\}, \{1,4,6\}, \{1,2\}, \{3,4,6\}, \{4,5\}, \{1,5\}$ is the 2-block dyadic shifted version of $X(n)$. The symbol $Td(2)$ denotes the 2- block dyadic shift operator. In the same manner, we obtain $Td(4)[Td(2)[X(n)]]=\{2,6\}, \{1,4,6\}, \{1,4,5\}, \{3,4,5\}, \{4,5\}, \{1,5\}, \{1,2\}, \{3,4,6\}$ and $Td(8)[Td(4)[Td(2)[X(n)]]]=\{1,4,6\}, \{2,6\}, \{3,4,5\}, \{1,4,5\}, \{1,5\}, \{4,5\}, \{3,4,6\}, \{1,2\}$. One can easily verify that all these dyadic shifted sequences have the same $Y(k)$, that is, $\{1,2,3,4,5,6\}, \{2,3\}, \{1,3,6\}, \{3,6\}, \{2,4,5,6\}, \{2\}, \{4,6\}, \{4,6\}$. There is yet another way of dyadic shifting input sequence $X(n)$ to $Td(2)[Td(4)[Td(8)[X(n)]]]$. Let us take $X(n) = \{1,2\}, \{3,4,6\}, \{4,5\}\{1,5\}, \{1,4,5\}, \{3,4,5\}, \{2,6\}, \{1,4,6\}$ and obtain following dyadic shifts: $Td(8)[X(n)]=\{3,4,6\}, \{1,2\}, \{1,5\}, \{4,5\}, \{3,4,5\}, \{1,4,5\}, \{1,4,6\}, \{2,6\}$ $Td(4)[Td(8)[X(n)]]=\{1,5\}, \{4,5\}, \{3,4,6\}, \{1,2\}, \{1,4,6\}, \{2,6\}, \{3,4,5\}, \{1,4,5\}$ and $Td(2)[Td(4)[Td(8)[X(n)]]]=\{1,4,6\}, \{2,6\}, \{3,4,5\}, \{1,4,5\}, \{1,5\}, \{4,5\}, \{3,4,6\}, \{1,2\}$. Note that $Td(2)[Td(4)[Td(8)[X(n)]]]=Td(8)[Td(4)[Td(2)[X(n)]]]$. One can easily verify from the above that other than $Td(4)[Td(2)[X(n)]]$ and $Td(8)[X(n)]$, all other dyadically permuted sequences fall under the category

of the cyclic permutation class of $X(n)$ and $X-1(n)$. This amounts to saying that the cyclic permutation class of $X(n)$ has eight non-repeating independent sequences, that of $X-1(n)$ has eight non-repeating independent sequences and the dyadic permutation classes of $X(n)$ has two non-repeating independent sequences. To conclude, all these 18 sequences could be seen to have the same $Y(k)$. Set Theoretic Rajan Transform has many emerging applications. It can be used as a powerful tool in encrypting digital (color) images. It has many other applications in domains like Signal Processing and Higher Order Mathematics.

III. APPLICATION OF STRT IN THE STUDY OF EXTENDED TOPOLOGICAL FILTERS DEFINED OVER A FINITE SET

Consider a finite set $X=\{a,b,c\}$. Then its power set is $\{\{\Phi\}, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$. One can construct a filter set F whose elements satisfy the following property: 'Any element of F ensures the presence of all its super sets present in the power set of X . For example consider a set $X = \{a,b,c\}$. The power set is $\{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$. The set $F = \{\{a\}, \{a,b\}, \{a,c\}, \{a,b,c\}\}$ is a valid topological filter set since every element in F ensures the presence of all its super sets. One can construct 18 such topological filters from the ground set X as shown in table 1.

Table 1: List of topological filters from $X = \{a,b,c\}$

Filters	Filter Contents	Cardinality
F1	$\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	7
F2	$\{\{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	6
F3	$\{\{a\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	6
F4	$\{\{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	6
F5	$\{\{a\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	5
F6	$\{\{b\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	5
F7	$\{\{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	5
F8	$\{\{a\}, \{a,b\}, \{a,c\}, \{a,b,c\}\}$	4
F9	$\{\{b\}, \{a,b\}, \{b,c\}, \{a,b,c\}\}$	4
F10	$\{\{c\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	4
F11	$\{\{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	4
F12	$\{\{a,b\}, \{a,c\}, \{a,b,c\}\}$	3
F13	$\{\{a,b\}, \{b,c\}, \{a,b,c\}\}$	3
F14	$\{\{a,c\}, \{b,c\}, \{a,b,c\}\}$	3
F15	$\{\{a,b\}, \{a,b,c\}\}$	2
F16	$\{\{a,c\}, \{a,b,c\}\}$	2
F17	$\{\{b,c\}, \{a,b,c\}\}$	2
F18	$\{\{a,b,c\}\}$	1

Lattice of topological filters

The lattice $\langle \Phi, \subseteq \rangle$ is constructed as given in Fig. 2 whose elements are 18 topological filters defined over the ground set $X = \{a, b, c\}$. Note that the symbol \subseteq denotes the partial order relation of 'subset of'.

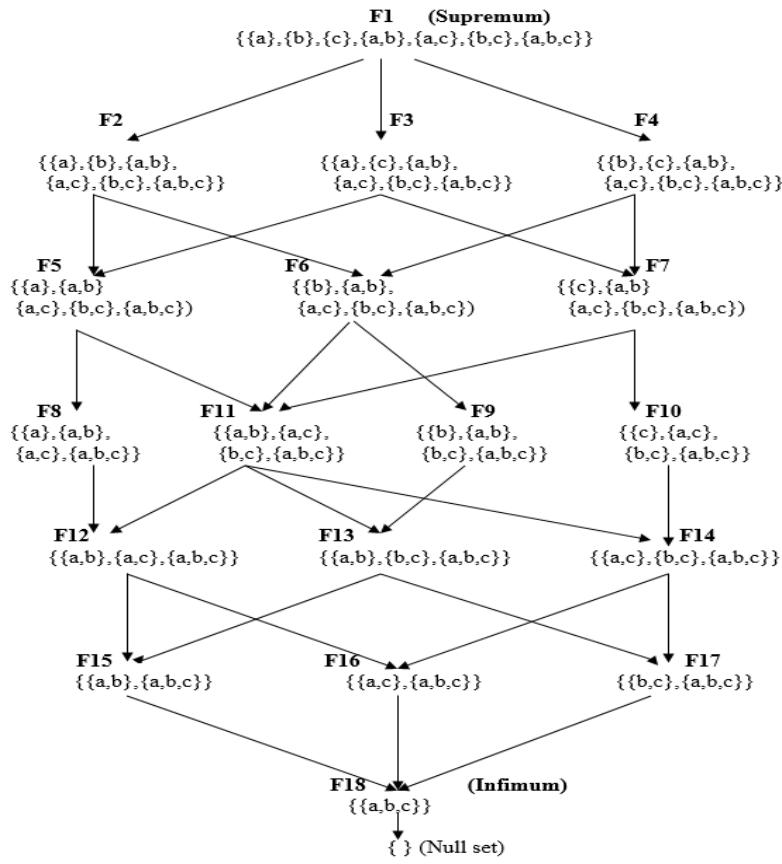


Fig. 2: Lattice diagram showing the linear filter chains over a set $X = \{a, b, c\}$

To apply STRT to a filter chain, the length of the chain should be a power of 2. The length of the chain in this case is 7. So the null set $\{\}$ is considered as the eighth filter as it is a subset of any set. By applying STRT

to these 48 filter chains, we get their corresponding spectra. Table 2 gives the STRT spectra of all 48 linear filter chains.

Table 2: STRT spectra of all 48 linear filter chains

Filter chain # 1 and its STRT spectrum

F1	$\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{a\},$ $, \{b,c\}, \{a,b,c\}\}$	$\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{a\},$ $, \{b,c\}, \{a,b,c\}\}$	$\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\},$ $, \{b,c\}, \{a,b,c\}\}$	$\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\},$ $, \{b,c\}, \{a,b,c\}\}$
F2	$\{\{a\}, \{b\}, \{a,b\}, \{a,c\},$ $, \{b,c\}, \{a,b,c\}\}$	$\{\{a\}, \{b\}, \{a,b\}, \{a,c\},$ $, \{b,c\}, \{a,b,c\}\}$	$\{\{a\}, \{b\}, \{a,b\}, \{a,c\},$ $, \{b,c\}, \{a,b,c\}\}$	$\{\{c\}\}$
F5	$\{\{a\}, \{a,b\}, \{a,c\}, \{b,c\},$ $, \{a,b,c\}\}$	$\{\{a\}, \{a,b\}, \{a,c\}, \{b,c\},$ $, \{a,b,c\}\}$	$\{\{b\}, \{c\}\}$	$\{\{b\}, \{c\}, \{b,c\}\}$
F8	$\{\{a\}, \{a,b\}, \{a,c\}, \{a,b,c\}$	$\{\{a\}, \{a,b\}, \{a,c\}, \{a,b,c\}$	$\{\{b\}, \{b,c\}\}$	$\{\{c\}, \{b,c\}\}$
F12	$\{\{a,b\}, \{a,c\}, \{a,b,c\}\}$	$\{\{a\}, \{b\}, \{c\}, \{b,c\}\}$	$\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\},$ $, \{b,c\}\}$	$\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\},$ $, \{b,c\}, \{a,b,c\}\}$
F15	$\{\{a,b\}, \{a,b,c\}\}$	$\{\{a\}, \{b\}, \{a,c\}, \{b,c\}\}$	$\{\{a\}, \{b\}, \{a,b\}, \{a,c\},$ $, \{b,c\}, \{a,b,c\}\}$	$\{\{c\}, \{a,b,c\}\}$
F18	$\{\{a,b,c\}\}$	$\{\{a\}, \{a,b\}, \{a,c\}, \{b,c\}\}$	$\{\{b\}, \{c\}, \{a,b\}, \{a,c\}\}$	$\{\{b\}, \{c\}, \{a,b\}, \{a,c\},$ $, \{b,c\}, \{a,b,c\}\}$
-	$\{\Phi\}$	$\{\{a\}, \{a,b\}, \{a,c\}, \{a,b,c\}\}$	$\{\{b\}, \{a,b\}, \{b,c\}, \{a,b,c\}\}$	$\{\{c\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

				{a,b,c}})
-	{Φ}	{c}, {a,c}, {b,c}, {a,b,c}}	{a}, {a,b}, {b,c}, {a,b,c}}	{b}, {a,b}, {a,c}, {a,b,c}}

Filter chain # 33 and its STRT spectrum

F1	{a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c})			
F4	{b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c})	{b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c})	{b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c})	{a}
F6	{b}, {a,b}, {a,c}, {b,c}, {a,b,c})	{b}, {a,b}, {a,c}, {b,c}, {a,b,c})	{a,c})	{a}, {b}, {c})
F11	{a,b}, {a,c}, {b,c}, {a,b,c})	{a,b}, {a,c}, {b,c}, {a,b,c})	{b}, {c})	{a}, {b})
F12	{a,b}, {a,c}, {a,b,c})	{a}, {b}, {c}, {b,c})	{a}, {b}, {c}, {a,b}, {a,c}, {b,c})	{a}, {b}, {c}, {a,b}, {a,c}, {b,c})
F15	{a,b}, {a,b,c})	{b}, {c}, {a,c}, {b,c})	{b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c})	{a}, {a,b,c})
F18	{a,b,c})	{b}, {a,b}, {a,c}, {b,c})	{a}, {c}, {a,b}, {a,c})	{a}, {b}, {c}, {a,b}, {a,c}, {b,c})
-	{Φ}	{a,b}, {a,c}, {b,c}, {a,b,c})	{b}, {c}, {a,b}, {a,b,c})	{a}, {b}, {a,c}, {a,b,c})

Filter chain # 34 and its STRT spectrum

F1	{a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c})			
F4	{b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c})	{b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c})	{b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c})	{a})
F6	{b}, {a,b}, {a,c}, {b,c}, {a,b,c})	{b}, {a,b}, {a,c}, {b,c}, {a,b,c})	{a,c})	{a}, {b}, {c})
F11	{a,b}, {a,c}, {b,c}, {a,b,c})	{a,b}, {a,c}, {b,c}, {a,b,c})	{b}, {c})	{a}, {b})
F12	{a,b}, {a,c}, {a,b,c})	{a}, {b}, {c}, {b,c})	{a}, {b}, {c}, {a,b}, {a,c}, {b,c})	{a}, {b}, {c}, {a,b}, {a,c}, {b,c})
F16	{a,c}, {a,b,c})	{b}, {c}, {a,b}, {b,c})	{b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c})	{a}, {a,b,c})
F18	{a,b,c})	{b}, {a,b}, {a,c}, {b,c})	{a}, {c}, {a,b}, {a,c})	{a}, {b}, {c}, {a,b}, {a,c}, {b,c})
-	{Φ}	{a,b}, {a,c}, {b,c}, {a,b,c})	{b}, {c}, {a,b}, {a,b,c})	{a}, {b}, {a,c}, {a,b,c})

Filter chain # 35 and its STRT spectrum

F1	{a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c})			
F4	{b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c})	{b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c})	{b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c})	{a})
F6	{b}, {a,b}, {a,c}, {b,c}, {a,b,c})	{b}, {a,b}, {a,c}, {b,c}, {a,b,c})	{a,c})	{a}, {b}, {c})
F11	{a,b}, {a,c}, {b,c}, {a,b,c})	{a,b}, {a,c}, {b,c}, {a,b,c})	{b}, {c})	{a}, {b})
F13	{a,b}, {b,c}, {a,b,c})	{a}, {b}, {c}, {a,c})	{a}, {b}, {c}, {a,b}, {a,c}, {b,c})	{a}, {b}, {c}, {a,b}, {a,c}, {b,c})
F15	{a,b}, {a,b,c})	{b}, {c}, {a,c}, {b,c})	{b}, {c}, {a,b}, {a,c}, {b,c})	{a}, {a,b,c})
F18	{a,b,c})	{b}, {a,b}, {a,c}, {b,c})	{a}, {c}, {a,b}, {b,c})	{a}, {b}, {c}, {a,b}, {b,c})

Filter chain # 46 and its STRT spectrum

F1	$\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$			
F4	$\{\{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{a\}\}$
F7	$\{\{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{a\}, \{b\}\}$	$\{\{a\}, \{b\}, \{c\}\}$
F11	$\{\{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{b\}, \{c\}\}$	$\{\{a\}, \{c\}\}$
F14	$\{\{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{a\}, \{b\}, \{c\}, \{a,b\}\}$	$\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$	$\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$
F17	$\{\{b,c\}, \{a,b,c\}\}$	$\{\{b\}, \{c\}, \{a,b\}, \{a,c\}\}$	$\{\{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{a\}, \{a,b,c\}\}$
F18	$\{\{a,b,c\}\}$	$\{\{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$	$\{\{a\}, \{b\}, \{a,c\}, \{b,c\}\}$	$\{\{a\}, \{b\}, \{c\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$
-	$\{\Phi\}$	$\{\{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{b\}, \{c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{a\}, \{c\}, \{a,c\}, \{a,b,c\}\}$

Filter chain # 47 and its STRT spectrum

F1	$\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$			
F4	$\{\{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{a\}\}$
F7	$\{\{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{a\}, \{b\}\}$	$\{\{a\}, \{b\}, \{a,b\}\}$
F10	$\{\{c\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{c\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{b\}, \{a,b\}\}$	$\{\{a\}, \{a,b\}\}$
F14	$\{\{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{a\}, \{b\}, \{c\}, \{a,b\}\}$	$\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$	$\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$
F16	$\{\{a,c\}, \{a,b,c\}\}$	$\{\{b\}, \{c\}, \{a,b\}, \{b,c\}\}$	$\{\{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{a\}, \{a,b,c\}\}$
F18	$\{\{a,b,c\}\}$	$\{\{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$	$\{\{a\}, \{b\}, \{a,c\}, \{b,c\}\}$	$\{\{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$
-	$\{\Phi\}$	$\{\{c\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{b\}, \{a,b\}, \{a,c\}, \{a,b,c\}\}$	$\{\{a\}, \{a,b\}, \{b,c\}, \{a,b,c\}\}$

Filter chain # 48 and its STRT spectrum

F1	$\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$			
F4	$\{\{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{a\}\}$
F7	$\{\{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{a\}, \{b\}\}$	$\{\{a\}, \{b\}, \{a,b\}\}$
F10	$\{\{c\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{c\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{b\}, \{a,b\}\}$	$\{\{a\}, \{a,b\}\}$
F14	$\{\{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{a\}, \{b\}, \{c\}, \{a,b\}\}$	$\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$	$\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$
F17	$\{\{b,c\}, \{a,b,c\}\}$	$\{\{b\}, \{c\}, \{a,b\}, \{a,c\}\}$	$\{\{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{a\}, \{a,b,c\}\}$
F18	$\{\{a,b,c\}\}$	$\{\{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$	$\{\{a\}, \{b\}, \{a,c\}, \{b,c\}\}$	$\{\{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$
-	$\{\Phi\}$	$\{\{c\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$	$\{\{b\}, \{a,b\}, \{b,c\}, \{a,b,c\}\}$	$\{\{a\}, \{a,b\}, \{a,c\}, \{a,b,c\}\}$

Consider a finite set $X = \{a, b, c, d\}$. Then its power set is $\{\{\}, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$. One can construct a filter set F whose elements satisfy the following property: 'Any element of F ensures the presence of all its super sets present in the power set of X '. One can construct 166 topological filters from the ground set $X = \{a, b, c, d\}$. This list is given in table 2.

Table 2: List of topological filters defined over $X = \{a, b, c, d\}$

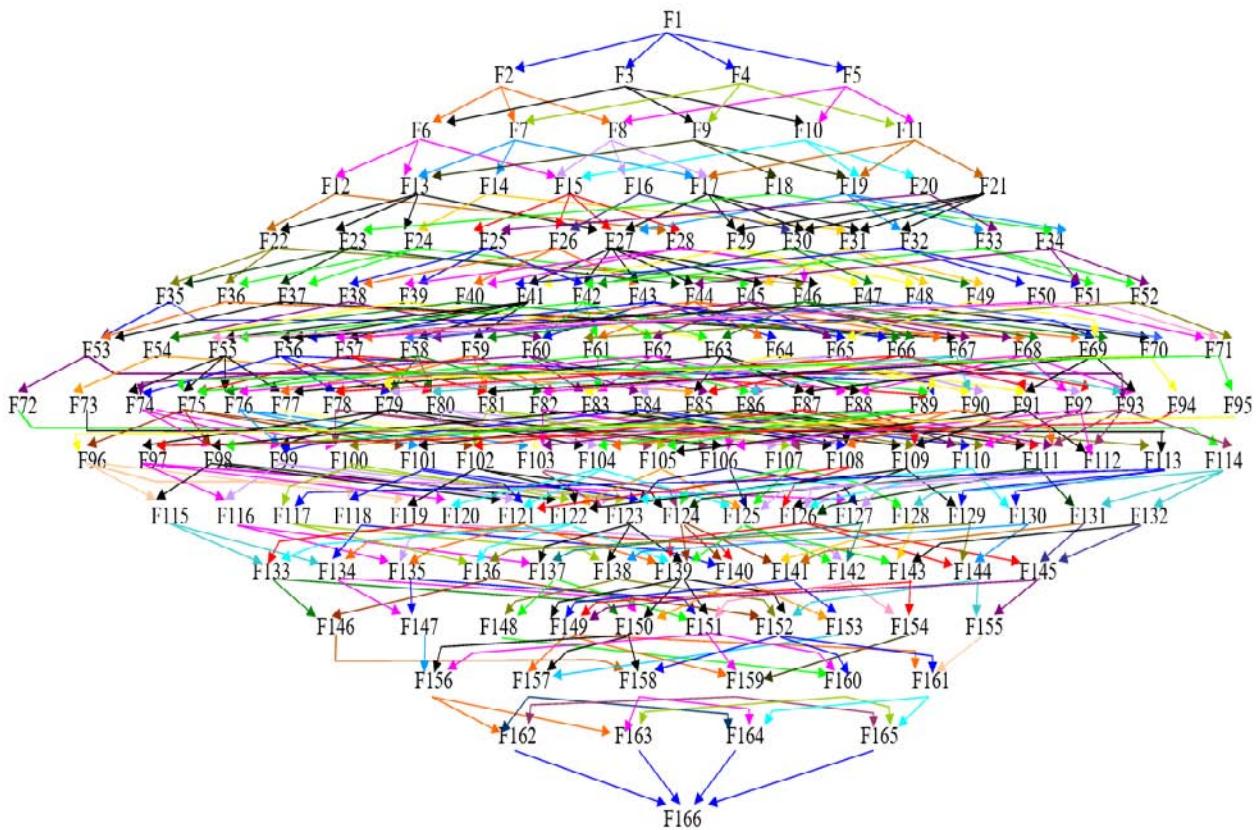
F57	$\{\{a,c\}, \{a,d\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	9
F58	$\{\{a,c\}, \{a,d\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	9
F59	$\{\{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	9
F60	$\{\{a,b\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	9
F61	$\{\{a,b\}, \{a,d\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	9
F62	$\{\{a,b\}, \{a,d\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	9
F63	$\{\{a,b\}, \{a,d\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	9
F64	$\{\{a,b\}, \{a,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	9
F65	$\{\{a,b\}, \{a,c\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	9
F66	$\{\{a,b\}, \{a,c\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	9
F67	$\{\{a,b\}, \{a,c\}, \{a,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	9
F68	$\{\{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	9
F69	$\{\{a,b\}, \{a,c\}, \{a,d\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	9
F70	$\{\{c\}, \{a,c\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	9
F71	$\{\{d\}, \{a,d\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	9
F72	$\{\{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{a,b,c,d\}\}$	8
F73	$\{\{b\}, \{a,b\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F74	$\{\{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F75	$\{\{a,d\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F76	$\{\{a,d\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F77	$\{\{a,d\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F78	$\{\{a,c\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F79	$\{\{a,c\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F80	$\{\{a,c\}, \{a,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F81	$\{\{a,c\}, \{a,d\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F82	$\{\{a,c\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F83	$\{\{a,b\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F84	$\{\{a,b\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F85	$\{\{a,b\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F86	$\{\{a,b\}, \{a,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F87	$\{\{a,b\}, \{a,d\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F88	$\{\{a,b\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F89	$\{\{a,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F90	$\{\{a,b\}, \{a,c\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F91	$\{\{a,b\}, \{a,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F92	$\{\{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F93	$\{\{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F94	$\{\{c\}, \{a,c\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F95	$\{\{d\}, \{a,d\}, \{b,d\}, \{c,d\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	8
F96	$\{\{a,d\}, \{b,d\}, \{c,d\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	7
F97	$\{\{a,c\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	7
F98	$\{\{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	7
F99	$\{\{b,c\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	7
F100	$\{\{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	7
F101	$\{\{a,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	7
F102	$\{\{a,d\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	7
F103	$\{\{a,d\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	7
F104	$\{\{a,c\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	7
F105	$\{\{a,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	7
F106	$\{\{a,c\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	7
F107	$\{\{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	7
F108	$\{\{a,b\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	7
F109	$\{\{a,b\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	7
F110	$\{\{a,b\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	7
F111	$\{\{a,b\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	7
F112	$\{\{a,b\}, \{a,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	7
F113	$\{\{a,b\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	7
F114	$\{\{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{a,b,c,d\}\}$	7
F115	$\{\{b,d\}, \{c,d\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	6
F116	$\{\{b,c\}, \{c,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	6

F117	$\{\{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	6
F118	$\{\{a,d\}, \{c,d\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	6
F119	$\{\{a,d\}, \{b,d\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	6
F120	$\{\{a,c\}, \{c,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	6
F121	$\{\{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	6
F122	$\{\{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	6
F123	$\{\{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	6
F124	$\{\{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	6
F125	$\{\{a,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	6
F126	$\{\{a,b\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	6
F127	$\{\{a,c\}, \{b,c\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	6
F128	$\{\{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{a,b,c,d\}\}$	6
F129	$\{\{a,b\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	6
F130	$\{\{a,b\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	6
F131	$\{\{a,b\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{a,b,c,d\}\}$	6
F132	$\{\{a,b\}, \{a,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{a,b,c,d\}\}$	6
F133	$\{\{b,d\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	5
F134	$\{\{c,d\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	5
F135	$\{\{c,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	5
F136	$\{\{b,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	5
F137	$\{\{b,c\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	5
F138	$\{\{b,c\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	5
F139	$\{\{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	5
F140	$\{\{a,d\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	5
F141	$\{\{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{a,b,c,d\}\}$	5
F142	$\{\{a,c\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	5
F143	$\{\{a,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{a,b,c,d\}\}$	5
F144	$\{\{a,b\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	5
F145	$\{\{a,b\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{a,b,c,d\}\}$	5
F146	$\{\{b,d\}, \{a,b,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	4
F147	$\{\{c,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	4
F148	$\{\{b,c\}, \{a,b,c\}, \{b,c,d\}, \{a,b,c,d\}\}$	4
F149	$\{\{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{a,b,c,d\}\}$	4
F150	$\{\{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	4
F151	$\{\{a,b,c\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	4
F152	$\{\{a,b,c\}, \{a,b,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	4
F153	$\{\{a,d\}, \{a,b,d\}, \{a,c,d\}, \{a,b,c,d\}\}$	4
F154	$\{\{a,c\}, \{a,b,c\}, \{a,c,d\}, \{a,b,c,d\}\}$	4
F155	$\{\{a,b\}, \{a,b,c\}, \{a,b,d\}, \{a,b,c,d\}\}$	4
F156	$\{\{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	3
F157	$\{\{a,b,d\}, \{a,c,d\}, \{a,b,c,d\}\}$	3
F158	$\{\{a,b,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	3
F159	$\{\{a,b,c\}, \{a,c,d\}, \{a,b,c,d\}\}$	3
F160	$\{\{a,b,c\}, \{b,c,d\}, \{a,b,c,d\}\}$	3
F161	$\{\{a,b,c\}, \{a,b,d\}, \{a,b,c,d\}\}$	3
F162	$\{\{a,c,d\}, \{a,b,c,d\}\}$	2
F163	$\{\{b,c,d\}, \{a,b,c,d\}\}$	2
F164	$\{\{a,b,d\}, \{a,b,c,d\}\}$	2
F165	$\{\{a,b,c\}, \{a,b,c,d\}\}$	2
F166	$\{\{a,b,c,d\}\}$	1

The lattice $\langle \Phi, \subseteq \rangle$ is constructed as given in Fig. 3 whose elements are 166 topological filters defined over the ground set $X = \{a,b,c,d\}$. Note that the symbol \subseteq denotes the partial order relation of 'subset of'. One can enumerate 13,767 linear maximal filter chains from this lattice. One can compute STRT spectra for all the 13,767 linear maximal filter chains. For example, one linear maximal filter chain is considered here and its STRT shown in table 3.

Table 3: Filter chain and its STRT spectrum

Sl.No.	Filter Chain	Spectrum
1.	F1 $\{\{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	$\{\{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$
	F2 $\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	$\{\{d\}\}$
	F7 $\{\{a\}, \{c\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	$\{\{b\}, \{c\}, \{d\}\}$
	F13 $\{\{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	$\{\{c\}, \{d\}\}$
	F22 $\{\{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	$\{\{b\}, \{c\}, \{d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{b,c,d\}\}$
	F36 $\{\{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	$\{\{b\}, \{b,c,d\}\}$
	F53 $\{\{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$	$\{\{b\}, \{c\}, \{d\}, \{b,c\}, \{b,d\}, \{b,c,d\}\}$
	F72 $\{\{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{a,b,c,d\}\}$	$\{\{c\}, \{d\}, \{b,d\}, \{b,c,d\}\}$
	F114 $\{\{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{a,b,c,d\}\}$	$\{\{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$
	F132 $\{\{a,b\}, \{a,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{a,b,c,d\}\}$	$\{\{d\}, \{a,b,c,d\}\}$
	F145 $\{\{a,b\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{a,b,c,d\}\}$	$\{\{b\}, \{c\}, \{d\}, \{a,b,c\}, \{a,b,d\}, \{a,b,c,d\}\}$
	F155 $\{\{a,b\}, \{a,b,c\}, \{a,b,d\}, \{a,b,c,d\}\}$	$\{\{c\}, \{d\}, \{a,b,c,d\}\}$
	F161 $\{\{a,b,c\}, \{a,b,d\}, \{a,b,c,d\}\}$	$\{\{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$
	F265 $\{\{a,b,c\}, \{a,b,c,d\}\}$	$\{\{d\}, \{a,d\}, \{b,c,d\}, \{a,b,c,d\}\}$
	F166 $\{\{a,b,c,d\}\}$	$\{\{b\}, \{c\}, \{d\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$
	{Φ}	$\{\{c\}, \{d\}, \{a,d\}, \{b,d\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$

Fig. 3: Lattice diagram showing the linear filter chains over a set $X = \{a, b, c, d\}$

IV. OBSERVATIONS

By applying STRT to the above maximal filter chains, we examined few pair-wise intersection properties. The level with filter of maximum cardinality is considered as Level1. By taking two random filter chains, which deviate at certain levels the following properties were observed:

Deviation in any combination of even levels results in following properties:

Union of spectra of two filter chains is same as Spectrum of Intersection of those two filter chains.

Intersection of spectra of two filter chains is same as Spectrum of Union of those two filter chains.

One can easily verify these properties by applying STRT to the below pair of filter chains:

For example, let us consider $n=3$,

Deviation in Level 2-

$$\begin{aligned} & F1-F2-F5-F8-F12-F15-F18 \\ & F1-F3-F5-F8-F12-F15-F18 \end{aligned}$$

Deviation in Level 4-

$$\begin{aligned} & F1-F3-F5-F8-F12-F15-F18 \\ & F1-F3-F5-F11-F12-F15-F18 \end{aligned}$$

Deviation in Level 6-

$$\begin{aligned} & F1-F4-F7-F10-F14-F16-F18 \\ & F1-F4-F7-F10-F14-F17-F18 \end{aligned}$$

Deviation in Level 4 and 6-

$$\begin{aligned} & F1-F2-F5-F8-F12-F15-F18 \\ & F1-F2-F5-F11-F12-F16-F18 \end{aligned}$$

Deviation in any combination of odd levels results in following properties:

Union of spectra of two filter chains is same as Spectrum of Union of those two filter chains.

Intersection of spectra of two filter chains is same as Spectrum of Intersection of those two filter chains.

One can easily verify these properties by applying STRT to the below pair of filter chains:

Deviation in Level 3-

$$\begin{aligned} & F1-F3-F5-F11-F12-F15-F18 \\ & F1-F3-F7-F11-F12-F15-F18 \end{aligned}$$

Deviation in Level 5-

$$\begin{aligned} & F1-F2-F5-F11-F12-F16-F18 \\ & F1-F2-F5-F11-F14-F16-F18 \end{aligned}$$

Deviation in Level 3 and 5-

$$\begin{aligned} & F1-F3-F5-F11-F12-F15-F18 \\ & F1-F3-F7-F11-F13-F15-F18 \end{aligned}$$

V. CONCLUDING REMARKS

All orthogonal transforms, be it continuous or discrete, are models of first order logic, that is, they have been developed in the framework of first order

logic that deal with elements of sets. Alternatively, STRT is a novel concept developed in the framework of second order logic that deals with set of sets, and so it has potential applications to solve problems related to functions of sets.

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