On The Propagation of Electric Pulse

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Abstract

Considering longitudinal flow in the axon plasma and introducing the substantial derivative, a nonlinear differential equation describing signal propagation along with the axon fiber instead of the cable equation is established. In contrast to the cable equation, which gives solutions that distort the profile of the pulse, a nonlinear equation describes a traveling wave of an unchangeable profile. The speed of the electric pulse wave, w = 320?a m/s, changes proportionally to the square root of the axon fiber radius, a.

Index terms— axon, propagation, action potential, differential equation.

1 Introduction

nalogous to the wires that connect different points in an electric circuit, axons are responsible for the transmission of information over the nervous system. The axon, as a part of a cell, separates the internal medium from the external one with the plasma membrane and any perturbations of a membrane potential (signal) conducted along the axon. The membrane potential is the result of ionic gradient currents perpendicular to the membrane.

The following events characterize the transmission of a nerve impulse (Figure 1):

? The resting potential describes the polarized state of a neuron (-70 mV). 1). A graded potential occurs in cell bodies and dendrites and is a local event that does not travel from its origin.

? Unlike a static graded potential, an action potential is dynamic and capable of traveling along with a nerve fiber. II.

2 Cable Equation

Nerve impulses flow in only one direction, accompanied by currents across and along the axon trunk. Current across the membrane is In turn, these ionic currents give rise to longitudinal currents that allow the regeneration of the membrane potential changes in the axon [1]:?? ?? = ?? ?? ???? ?? ???? , where V m = V in -V out is the membrane potential , ?? ?? = ???? 0 2??????/???? ?? = ? ???? ?? ?? ???? , **(1)**

where x is the axis along the axon, ?? ?? = ?? ?? ???? 2 is the resistance per unit length of the axon plasma.

The propagation of the action potential is described by the well-known cable differential equation [1][2][3]:?? ???? ???? = ?? 2 ?? 2 ?? ???? 2 + ???,**(2)**

where ? = ? ab? m /2? i is the space constant and ? = k? 0 ? m is the time constant,??? = ?? ???????? -?? ???????? = ?70????.

The evolution of any initial impulse ??(???, 0) is given by the integral??(??, ??) = ???{ ?? ?? + ? ?? 4???? ? ??(???,0) ?????? ?????? [? ???????? ? ? 2 4???? 2]?????} ? 0 . (3)

The nerve impulse describing by this equation changes its form soon during propagation along with the nerve fiber. For example, for the evolution of ?-shape initial impulse, ??(??', 0) = ???????(?? ?) integration leads to??(??, ??) = ???[?? ?? + ? ?? 4???? ?????? (? ???? 2 4???? 2)],**(4)**

that describes an ink drop diffusion in water, hence the nerve impulse spreads in space-time, distorting initial form, therefore the transmission information. Therefore, the transition of information through the nerve fiber should be occurring by a pulse-wave mechanism that does not change its initial shape.

3 III.

A New Equation for the Signal Transmission ?? ???? ???? ? ???? 2 ? ???? ???? ? 2 = ?? 2 ?? 2 ?? ???? 2 + ???**(5)**

where the constant parameter ? = ??? i U i /? 2 ?? i characterizes the longitudinal flow of substance. Introduction of dimensionless running wave variable ? = (x -wt)/?, where w is the speed of transmission of an electric signal (potential change) over the nervous system, gives ordinary nonlinear differential equation for perturbed potential???? + ???? ? 2 + (???? ??)??? + ??? = 0,**(6)**

where prime means derivative with respect ?.

The general solution of nonlinear equation (4), considering ?? = ??/?? is:1 1 - ln 1 4 ( ) 2**(**
2)2 1 V C Cosh V C ? ? ? ? ? ? ? = + ? ? + . (**7**)

For the intensity of electric field corresponding to generated potential (5), we obtain1 1 ( ) [1 1-4 1-4 ( )] 2 2 1 dV E V Tanh V C dx ? ? ? ? ? ? = ? = ? ? ? +**(8)**

Taking into account the following boundary conditions:V(0) = V peak and V'(0) = 0,**(9)**

For constants C 1 and C 2 we obtain:

On The Propagation of Electric Pulse . 41 1 2 peak C V V V C V V V ? ? ? ? ? ? = ? ? ? ? =? ? ? ? ?

With the use of numerical values of parameters [4,5], we can establish the profile of the electric intensity in the running wave (Fig. 2).

4 Conclusion

Considering the longitudinal flow in the electrolyte of an axon plasma and introducing the substantial derivative, we have established nonlinear differential equation (3) describing signal propagation along with the axon fiber as a running wave without changing its profile in contrast to cable equation (2). The solution to this equation is obtained in the form of a running wave. The speed of this wave is

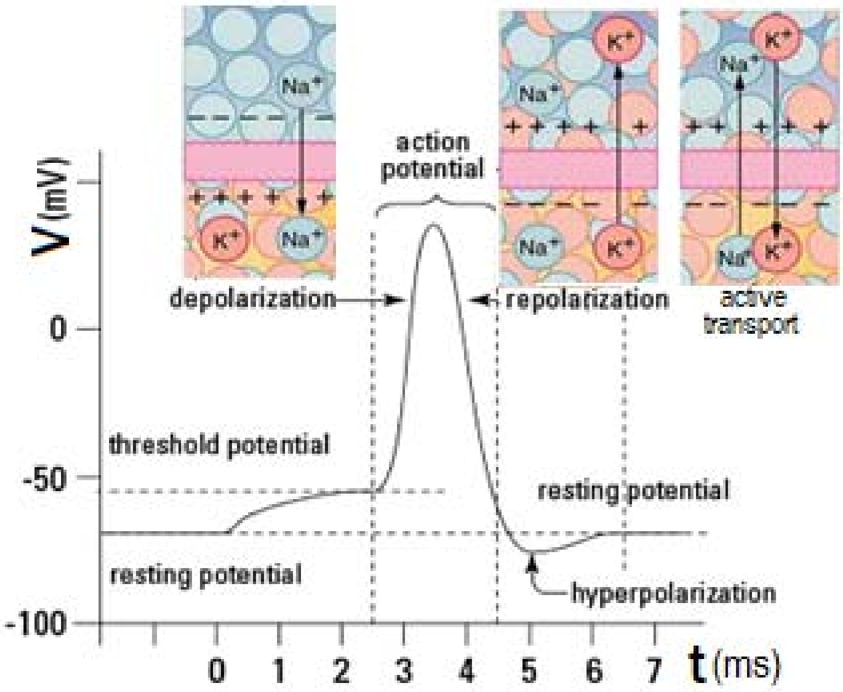

Figure 1:

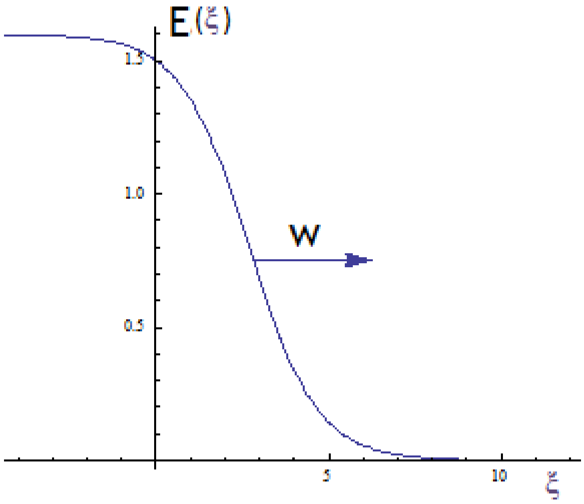

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Figure 2: Figure 1 :

[Note: + channels open, then positive potassium ions exit across the membrane and hyperpolarizes it (Figure]

Figure 3:

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