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On The Propagation of Electric Pulse

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I. Introduction

Analogous to the wires that connect different points in an electric circuit, axons are responsible for the transmission of information over the nervous system. The axon, as a part of a cell, separates the internal medium from the external one with the plasma membrane and any perturbations of a membrane potential (signal) conducted along the axon. The *membrane potential* is the result of ionic gradient currents perpendicular to the membrane.

The following events characterize the transmission of a nerve impulse (Figure 1):

- The resting potential describes the polarized state of a neuron (–70 mV).
- *A graded potential* is a change in the resting potential in the response to a stimulus, which causes Na^+ or K^+ gated channels to open. If Na^+ channels open, the membrane *depolarizes*. If the K^+ channels open, then positive potassium ions exit across the membrane and *hyperpolarizes it (Figure 1).* A graded potential occurs in cell bodies and dendrites and is a local event that does not travel from its origin.

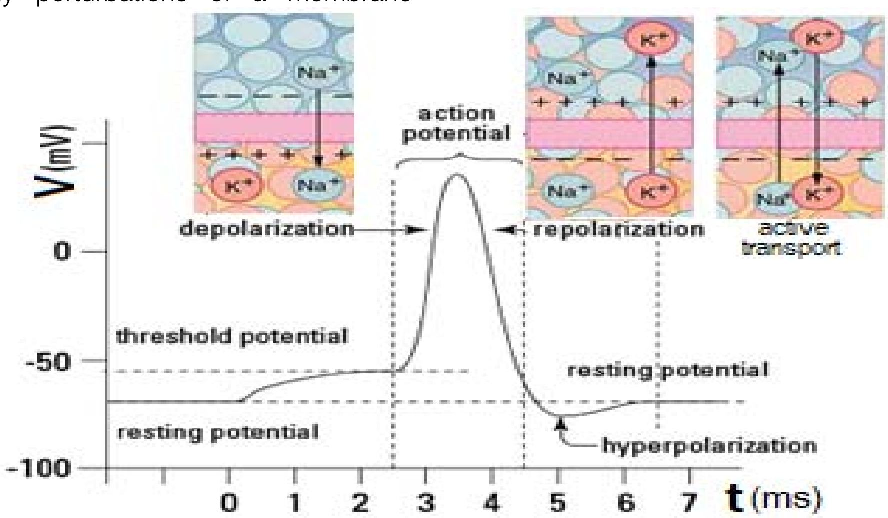

Figure 1: Events that characterize the transmission of a nerve impulse

- Unlike a static graded potential, an *action potential* is dynamic and capable of traveling along with a nerve fiber. If a graded potential is sufficiently large, Na^+ channels open, and Na^+ on the outside of the membrane becomes depolarized (-70 to +30 mV). If the stimulus exceeds a certain *threshold level* (~-55 mV) - additional Na^+ gates open, increasing the flow of Na^+, causing an action potential, which stimulates neighboring Na^+ gates to open. In this manner, the action potential travels down the length of the axon as opened Na^+ gates stimulate neighboring Na^+ gates to open…
- The inflow of Na^+, K^+ channels open, allowing K^+ movement out of the cell causes *repolarization* by restoring the original membrane polarization by Na^+/K^+ pumps in the cell membrane.

II. Cable Equation

Nerve impulses flow in only one direction, accompanied by currents across and along the axon trunk. Current across the membrane is

$$I_m = C_m \frac{dV_m}{dt},$$

where $V_m = V_{in} - V_{out}$ is the membrane potential, $C_m = k\varepsilon_0 2\pi a L / b$ is the capacity of an unmyelinated

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cylindrical axon with a length of L (=1m), a membrane thickness of *b* = 10 nm, and a radius of *a* = 2.5 μm. The length of the myelinated section is 1.4 mm and the thickness is b = 2μm.

In turn, these ionic currents give rise to longitudinal currents that allow the regeneration of the membrane potential changes in the axon [1]:

$$I_i = -\frac{dV}{r_i dx}, \qquad (1)$$

where x is the axis along the axon, $r_i = \frac{\rho_i}{\pi a^2}$ is the resistance per unit length of the axon plasma.

The propagation of the *action potential* is described by the well-known cable differential equation [1-3]:

$$\tau\frac{\partial V}{\partial t} = \lambda^2\frac{\partial^2 V}{\partial x^2} + \Delta V, \qquad (2)$$

where $\lambda = \sqrt{ab\rho_m/2\rho_i}$ is the space constant and $\tau = k\varepsilon_0\rho_m$ is the time constant,

$$\Delta V = V_{peak} - V_{rest} = -70mV.$$

The evolution of any initial impulse $V(x',0)$ is given by the integral

$$V(v,t) = \Delta V\{\frac{t}{\tau} + \sqrt{\frac{\tau}{4\pi t}}\int_0^\infty \frac{V(x',0)}{\Delta V\lambda} exp\,[-\frac{\tau(x-x')^2}{4t\lambda^2}]dx'\}. \qquad (3)$$

The nerve impulse describing by this equation changes its form soon during propagation along with the nerve fiber. For example, for the evolution of δ-shape initial impulse, $V(x',0) = \Delta V\lambda\delta(x')$ integration leads to

$$V(x,t) = \Delta V[\frac{t}{\tau} + \sqrt{\frac{\tau}{4\pi t}}\,exp\,(-\frac{\tau x^2}{4t\lambda^2})], \qquad (4)$$

that describes an ink drop diffusion in water, hence the nerve impulse spreads in space-time, distorting initial form, therefore the transmission information. Therefore, the transition of information through the nerve fiber should be occurring by a pulse-wave mechanism that does not change its initial shape.

III. A New Equation for the Signal Transmission

Because of longitudinal current, a flow of substance takes place along the axon plasma with an effective speed

$$u = -\left(\frac{\Sigma\mu_i U_i}{\Sigma\mu_i}\right)\frac{\partial V}{\partial x},$$

where μ_i is the molar mass of i^{th} type ion and U_i is corresponding mobility. The mass flow in the axon plasma requires the use of the substantial derivative,

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x},$$

instead of partial $\frac{\partial}{\partial t}$, in the left-hand side of the cable equation (2).

$$\tau\frac{\partial V}{\partial t} - \gamma\lambda^2\left(\frac{\partial V}{\partial x}\right)^2 = \lambda^2\frac{\partial^2 V}{\partial x^2} + \Delta V \qquad (5)$$

where the constant parameter $\gamma = \tau\Sigma\mu_i U_i/\lambda^2\Sigma\mu_i$ characterizes the longitudinal flow of substance.

Introduction of dimensionless running wave variable $\xi = (x - wt)/\lambda$, where w is the speed of transmission of an electric signal (potential change) over the nervous system, gives ordinary nonlinear differential equation for perturbed potential

$$V'' + \gamma V'^2 + (\frac{w\tau}{\lambda})V' + \Delta V = 0, \qquad (6)$$

where prime means derivative with respect ξ.

The general solution of nonlinear equation (4), considering $w = \lambda/\tau$ is:

$$V(\xi) = C_2 - \frac{\xi}{2\gamma} + \frac{1}{\gamma}\ln Cosh\frac{1}{2}\sqrt{1-4\gamma\Delta V}(\xi + C_1) \qquad (7)$$

For the intensity of electric field corresponding to generated potential (5), we obtain

$$E(\xi) = -\frac{dV}{dx} = \frac{1}{2\gamma}[1-\sqrt{1-4\gamma\Delta V}Tanh\frac{1}{2}\sqrt{1-4\gamma\Delta V}(\xi + C_1)] \qquad (8)$$

Taking into account the following boundary conditions:

$$V(0) = V_{peak} \text{ and } V'(0) = 0, \qquad (9)$$

For constants C_1 and C_2 we obtain:

$$C_1 = \frac{2}{\sqrt{1-4\gamma\Delta V}}\text{Tanh}^{-1}\frac{1}{\sqrt{1-4\gamma\Delta V}},$$

$$C_2 = \frac{V_{peak}}{\Delta V} - \frac{1}{\gamma\Delta V}\ln\sqrt{1-\frac{1}{4\gamma\Delta V}}.$$

With the use of numerical values of parameters [4, 5], we can establish the profile of the electric intensity in the running wave (Fig. 2).

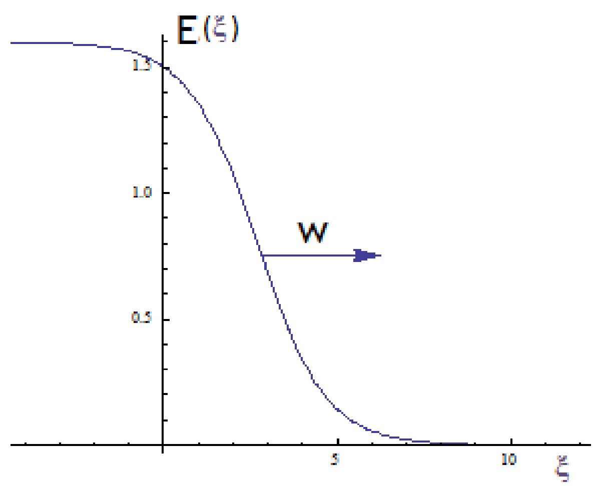

Figure 2: Profile of electric pulse propagation along an axon fiber

IV. Conclusion

Considering the longitudinal flow in the electrolyte of an axon plasma and introducing the substantial derivative, we have established nonlinear differential equation (3) describing signal propagation along with the axon fiber as a running wave without changing its profile in contrast to cable equation (2). The solution to this equation is obtained in the form of a running wave. The speed of this wave is

$$w = \frac{\lambda}{\tau} = \sqrt{\frac{ab}{\rho_i \rho_m k^2 \varepsilon_0^2}} = 320\sqrt{a}\,\frac{m}{s}, \qquad (10)$$

which changes proportionally to the square root of the axon fiber radius, *a*.

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