Abstract- The information available to the system is incomplete in many applications, particularly in Decision Support Systems. The fuzzy logic deals incomplete information with belief rather than likelihood (probability). Sometimes the decision has to be taken with fuzzy information. In this paper, fuzzy machine learning is studied for decision support systems. The fuzzy Decision set is defined with two-fold fuzzy set. The fuzzy inference is studied with fuzzy neural network for fuzzy Decision sets. Business application is given as application.

Keywords: component, formatting, style, styling.

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Fuzzy Reinforcement Learning using Neural Network: An Application to Medical Diagnosis and Business Intelligence

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Abstract: The information available to the system is incomplete in many applications, particularly in Decision Support Systems. The fuzzy logic deals with incomplete information with belief rather than likelihood (probability). Sometimes the decision has to be taken with fuzzy information. In this paper, fuzzy machine learning is studied for decision support systems. The fuzzy Decision set is defined with two-fold fuzzy set. The fuzzy inference is studied with fuzzy neural network for fuzzy Decision sets. Business application is given as application.

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1. Introduction

Information available to many applications like Business, Medical, Geological, Control Systems, etc is incomplete or uncertain. The fuzzy logic will deal with incomplete information with belief rather than likelihood (probable). Zadeh formulated uncertain information as fuzzy set with a single membership function. The fuzzy set with two membership functions will give more evidence than a single membership function. The two-fold fuzzy set is with fuzzy membership functions “Belief” and “Disbelief”. Usually, in Medical and Business applications, there are two opinions like “Belief” and “Disbelief” about the information and decision has to be taken under risk. For instance, in Mycin[1], the medical information is defined with belief and disbelief i.e./, CF[h,e]=MB[h,e] – MD[h,e], where “e” is the evidence for given hypothesis “h”. The fuzzy set is used instead of Probability to define fuzzy certainty factor.

The fuzzy neural networks are one of the learning techniques to study fuzzy problems. In the following, some methods of fuzzy conditional inference are studied through fuzzy neural network and before that preliminaries of fuzzy logic and neural network are discussed.

In the following fuzzy logic [10] and Generalized fuzzy logic [9] are studied briefly. The fuzzy Certainty Factor is studied and fuzzy Decision set is proposed. The fuzzy inference and fuzzy reasoning are studied for fuzzy Decision set. The Business applications are studied as applications of fuzzy Decision set.

II. Fuzzy Logic

Various theories are studied to deal with imprecise, inconsistent and inexact information and these theories deal with likelihood whereas fuzzy logic with belief. Zadeh [10] has introduced fuzzy set as a model to deal with uncertain information as single membership functions. The fuzzy set is a class of objects with a continuum of grades of membership. The set A of X is characterized by its membership function \( \mu_A(x) \) and ranging values in the unit interval \([0, 1] \).

\[ \mu_A(x) : X \rightarrow [0, 1], x \in X, \text{where } X \text{ is Universe of discourse.} \]

\[ A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots + \mu_A(x_n)/x_n, \text{ “+” is union} \]

For example, the fuzzy proposition “x is young”

\[ \text{Young} = \{0.95/10 + 0.9/20 + 0.8/30 + 0.6/40 + 0.4/50 + 0.3/60 + 0.2/70 + 0.15/80 + 0.1/90 \} \]

Not young = \{0.05/10 + 0.1/20 + 0.2/30 + 0.4/40 + 0.6/50 + 0.8/60 + 0.7/70 + 0.95/80 + 0.9/90 \}

For instance “Rama is young” and the fuzziness of “young” is 0.8 The Graphical representation of young and not young is shown in fig.1

![Fig. 1: fuzzy membership function](image)

The fuzzy set of type 2 “Headache” is defined as

\[ \text{Headache} = \{0.4/mild + 0.6/moderate + 0.8/Serious\} \]

For example, consider the fuzzy proposition “x has mild Headache”
For instance “Rama has mild headache” with Fuzziness 0.4

The fuzzy logic is defined as combination of fuzzy sets using logical operators [21]. Some of the logical operations are given below

Let A, B and C are fuzzy sets. The operations on fuzzy sets are

**Negation**

\[
\text{If } x \text{ is not } A \\
A' = 1 - \mu_A(x) / x
\]

**Conjunction**

\[
x \text{ is } A \text{ and } y \text{ is } B \rightarrow (x, y) \text{ is } A \times B \\
A \times B = \min\{\mu_A(x), \mu_B(y)\}(x, y)
\]

\[
\text{If } x = y \\
x \text{ is } A \text{ and } y \text{ is } B \rightarrow (x, y) \text{ is } A \Lambda B \\
A \Lambda B = \min\{\mu_A(x), \mu_B(y)\}/x \text{ x is A or y is B} \rightarrow (x, y) \text{ is } A' \times B' \\
A' \times B' = \max\{\mu_A(x), \mu_B(y)\}(x, y)
\]

\[
\text{If } x = y \\
x \text{ is } A \text{ and } x \text{ is } B \rightarrow (x, x) \text{ is } A \Lambda B \\
A \Lambda B = \max\{\mu_A(x), \mu_B(y)\}/x
\]

**Implication**

\[
\text{If } x \text{ is } A \text{ then } y \text{ is } B \rightarrow A \rightarrow B = \min\{1, 1 - \mu_A(x) + \mu_B(y)\}/(x, y)
\]

\[
\text{if } x = y \\
A \rightarrow B = \min\{1, 1 - \mu_A(x) + \mu_B(y)\}/x
\]

\[
\text{If } x \text{ is } A \text{ then } y \text{ is } B \text{ else } y \text{ is } C \rightarrow (x, y) \\
A \rightarrow B = \min\{1, 1 - \mu_A(x) + \mu_B(y)\}/(x, y)
\]

\[
\text{if } x = y \\
A \rightarrow C = \min\{1, 1 - \mu_A(x) + \mu_C(y)\}/(x, y)
\]

**Composition**

\[
A \circ B = A \times B = \min\{\mu_A(x), \mu_B(y)\}(x, y)
\]

\[
\text{If } x = y \\
A \circ B = \min\{\mu_A(x), \mu_B(y)\}/x
\]

The fuzzy propositions may contain quantifiers like “Very”, “More or Less”. These fuzzy quantifiers may be eliminated as

**Concentration**

\[
\text{x is very } A \\
\mu_{\text{very, } A}(x) = \mu_A(x)^2
\]

**Diffusion**

\[
\text{x is very } A \\
\mu_{\text{more or less, } A}(x) = \mu_A(x)^{0.5}
\]

### III. Generalized Fuzzy Logic with two-Fold Fuzzy Set

Since formation of the generalized fuzzy set simply as two-fold fuzzy set and is extension Zadeh fuzzy logic.

The fuzzy logic is defined as combination of fuzzy sets using logical operators. Some of the logical operations are given below
Suppose A, B and C are fuzzy sets. The operations on fuzzy sets are given below for two-fold fuzzy sets.

Since formation of the generalized fuzzy set simply as two-fold fuzzy set, Zadeh fuzzy logic is extended to these generalized fuzzy sets.

Negation

\[ A' = \{ 1 - \mu_A^{\text{Belief}}(x), 1 - \mu_A^{\text{Disbelief}}(x) \} / x \]

Disjunction

\[ A \lor B = \{ \max(\mu_A^{\text{Belief}}(x) , \mu_A^{\text{Belief}}(y)), \max(\mu_B^{\text{Belief}}(x) , \mu_B^{\text{Belief}}(y)) \} / (x,y) \]

Conjunction

\[ A \land B = \{ \min(\mu_A^{\text{Belief}}(x) , \mu_A^{\text{Belief}}(y)), \min(\mu_B^{\text{Disbelief}}(x) , \mu_B^{\text{Disbelief}}(y)) \} / (x,y) \]

Implication

\[ A \rightarrow B = \{ \min (1, 1 - \mu_A^{\text{Belief}}(x) + \mu_B^{\text{Belief}}(y) , \min (1, 1 - \mu_A^{\text{Disbelief}}(x) + \mu_B^{\text{Disbelief}}(y)) / (x,y) \}

If x is A then y is B else y is C = A x B + A' x C

If x is A then y is B else y is C = A \rightarrow B = \{ \min (1, 1 - \mu_A^{\text{Belief}}(x) + \mu_B^{\text{Belief}}(y) , \min (1, 1 - \mu_A^{\text{Disbelief}}(x) + \mu_B^{\text{Disbelief}}(y)) / (x,y) \}

If x is not A then y is B else y is C = A' \rightarrow C = \min (1, \mu_A^{\text{Belief}}(x) + \mu_C^{\text{Belief}}(y) , \min (1, \mu_A^{\text{Disbelief}}(x) + \mu_C^{\text{Disbelief}}(y)) / (x,y)

Composition

\[ A \circ R = \{ \min_x (\mu_A^{\text{Belief}}(x) , \mu_A^{\text{Belief}}(x) ) , \min_x(\mu_R^{\text{Disbelief}}(x) , \mu_R^{\text{Disbelief}}(x) ) \} / y \]

The fuzzy propositions may contain quantifiers like "very", "more or less". These fuzzy quantifiers may be eliminated as

Concentration

\[ "x \text{ is very } A" \]

\[ \mu_{\text{very}A}(x) = \{ \mu_A^{\text{Belief}}(x)^2, \mu_A^{\text{Disbelief}}(x)^2 \} \]

Diffusion

\[ "x \text{ is more or less } A" \]

\[ \mu_{\text{more or less}A}(x) = \{ \mu_A^{\text{Belief}}(x)^0.5, \mu_A^{\text{Disbelief}}(x)^0.5 \} \]

For instance, Let A, B and C are

\[ A = \{ 0.8/x_1 + 0.9/x_2 + 0.7/x_3 + 0.6/x_4 + 0.5/x_5 , 0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.7/x_4 + 0.6/x_5 \} \]

\[ B = \{ 0.9/x_1 + 0.7/x_2 + 0.8/x_3 + 0.5/x_4 + 0.6/x_5 , 0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 + 0.7/x_5 \} \]

\[ A \lor B = \{ 0.9/x_1 + 0.9/x_2 + 0.8/x_3 + 0.6/x_4 + 0.6/x_5 , 0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.7/x_4 + 0.7/x_5 \} \]

\[ A \land B = \{ 0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 + 0.5/x_5 , 0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 + 0.6/x_5 \} \]

\[ A' = \text{ not } A = \{ 0.2/x_1 + 0.1/x_2 + 0.3/x_3 + 0.4/x_4 + 0.5/x_5 , 0.6/x_1 + 0.7/x_2 + 0.6/x_3 + 0.3/x_4 + 0.4/x_5 \} \]

\[ A \rightarrow B = \{ 1/x_1 + 0.8/x_2 + 1/x_3 + 0.9/x_4 + 1/x_5 , 1/x_1 + 1/x_2 + 1/x_3 + 0.8/x_4 + 1/x_5 \} \]

\[ A \circ B = \{ 0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 + 0.5/x_5 , 0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 + 0.6/x_5 \} \]

\[ \mu_{\text{Very}A}(x) = \{ \mu_A^{\text{Belief}}(x)^2, \mu_A^{\text{Disbelief}}(x)^2 \} \]

\[ = \{ 0.64/x_1 + 0.81/x_2 + 0.49/x_3 + 0.36/x_4 + 0.25/x_5 \} \]
The neural network concept is taken from the Biological activity of nervous system. The neurons passes information to other neurons. There are many models described for neural networks. The McCulloch-Pitts model contributed in understanding neural network and Zedeh explain that activity of neuron is fuzzy process [13].

The McCulloch and Pitt’s model consist of set of inputs, processing unit and output and it is shown in Fig. 2

![McCulloch and Pitt’s model](image1)

**Fig. 2:** McCulloch and Pitt’s model

The fuzzy neuron model for fuzzy conditional inference for

If \( x_1 \) is \( A_1 \) and/or \( x_2 \) is \( A_2 \) and/or … and/or \( x_n \) is \( A_n \) then \( B \) may be defined as set of individuals of the universe of discourse, fuzziness and computational functional function and shown in Fig. 3.

Where \( B = f(A_1, A_2, ..., A_n) \)

This fuzzy neuron fit for where the relation between president part and consequent part of fuzzy conditional inference is not known

![Fuzzy neuron model](image2)

**Fig. 3:** Fuzzy neuron model

The multilayer fuzzy neural net work is shown in Fig. 3

The fuzzy neuron for Defuzzification for Centre of Gravity (COG) is shown in fig. 4

![Defuzzification](image3)

**Fig. 4:** Defuzzification
V. Fuzzy Decision Set

Zadeh [10] proposed fuzzy set to deal with incomplete information. Generalized fuzzy set with two-fold membership function $\mu_A(x) = \{ \mu_A^{\text{Belief}}(x), \mu_A^{\text{Disbelief}}(x) \}$ is studied [18].

The fuzzy Certainty Factor may be defined as (FCF)

$$\mu_A^{\text{FCF}}(x) = \mu_A^{\text{Belief}}(x) - \mu_A^{\text{Disbelief}}(x),$$

where

$$0 \leq \mu_A^{\text{Belief}}(x) < \mu_A^{\text{Disbelief}}(x) \leq 1$$

Fuzzy Decision set R is defined based on convex fuzzy set [10]

$$R = \{ A, \mu_A^{\text{FCF}}(x) \geq \alpha \}, \text{ where } \alpha \in [0, 1]$$

For instance,

$$\text{Demand} = \{ 0.8/x1 + 0.7/x2 + 0.86/x3 + 0.75/x4 + 0.88/x5, 0.2/x1 + 0.3/x2 + 0.25/x3 + 0.3/x4 + 0.2/x5 \}$$

$$\mu_{\text{Demand}}^{\text{FCF}}(x) = 0.6/x1 + 0.4/x2 + 0.61/x3 + 0.45/x4 + 0.68/x5$$

The Generalized fuzzy set for Demand for the Items and fuzzy certainty factor is shown in Fig 5.

Suppose fuzzy Decision set is defined

$$\mu_{\text{Demand}}^{\text{FCF}}(x) \geq 0.5$$

$$= 1/x1 + 0/x2 + 1/x3 + 0/x4 + 1/x5$$

Decision may be taken under Decision shown in Fig 6.

The fuzzy logic is combination of logical operators. Consider the logical operations on fuzzy Decision sets r1, R2 and R3

Negation

If x is not R1
Conjunction

\[ R_1' = 1 - \mu_{R_1}(x)/x \]

\[ x \text{ is } R_1 \text{ and } y \text{ is } R_2 \rightarrow (x, y) \text{ is } R_1 \times R_2 \]

\[ R_1 \times R_2 = \min(\mu_{R_1}(x)), \mu_{R_2}(y)) \{x, y\} \]

If \( x = y \)

\[ x \text{ is } R_1 \text{ and } y \text{ is } R_2 \rightarrow (x, y) \text{ is } R_1 \land R_2 \]

\[ R_1 \land R_2 = \min(\mu_{R_1}(x)), \mu_{R_2}(y)) \{x, y\} \]

If \( x = y \)

\[ x \text{ is } R_1 \text{ and } x \text{ is } R_2 \rightarrow (x, x) \text{ is } R_1 \lor R_2 \]

\[ R_1 \lor R_2 = \max(\mu_{R_1}(x)), \mu_{R_2}(y)) \{x, y\} \]

Implication

\[ \text{if } x \text{ is } R_1 \text{ then } y \text{ is } R_2 \rightarrow \text{R}_1 \rightarrow \text{R}_2 = \min\{1, 1 - \mu_{R_1}(x) + \mu_{R_2}(y)\} \{x, y\} \]

if \( x = y \)

\[ \text{R}_1 \rightarrow \text{R}_2 = \min\{1, 1 - \mu_{R_1}(x) + \mu_{R_2}(y)\} \{x\} \]

Composition

\[ R_1 \circ R_2 = R_1 \times R_2 = \min(\mu_{R_1}(x)), \mu_{R_2}(y)) \{x, y\} \]

If \( x = y \)

\[ R_1 \circ R_2 = \min(\mu_{R_1}(x)), \mu_{R_2}(y)) \{x\} \]

The fuzzy propositions may contain quantifiers like “Very”, “More or Less”. These fuzzy quantifiers may be eliminated as

Concentration

\[ x \text{ is very } R_1 \]

\[ \mu_{\text{very } R_1}(x)) = \mu_{R_1}(x)^2 \]

Diffusion

\[ x \text{ is very } R_1 \]

\[ \mu_{\text{more or less } R_1}(x) = \mu_{R_1}(x)^{0.5} \]

Zadeh[10] fuzzy conditional inference is give as

If \( x_1 \) is \( R_1 \) and \( x_2 \) is \( R_2 \) and … and \( x_n \) is \( R_n \) then \( y \) is \( B \) = \( f(R_1, R_2, ..., R_n, B) = \min(1, 1 - R_1 + R_2 + ... + R_n + B) \)

The fuzzy neural network for Zadeh is shown in fig.7

Fig. 7: Zadeh fuzzy conditional inference
Mamdani [2] fuzzy conditional inference is given as
If \( x_1 \) is \( R_1 \) and \( x_2 \) is \( R_2 \) and … and \( x_n \) is \( R_n \) then \( y \) is \( B = f(R_1, R_2, \ldots, R_n, B) = \min(R_1, R_2, \ldots, R_n) \)

The fuzzy neural network for Mamdani is shown in fig. 8

Reddy [8] fuzzy conditional inference is given as
If \( x_1 \) is \( R_1 \) and \( x_2 \) is \( R_2 \) and … and \( x_n \) is \( R_n \) then \( y \) is \( B = f(R_1, R_2, \ldots, R_n, B) = \min(R_1, R_2, \ldots, R_n) \)

The fuzzy neural network for Reddy is shown in fig. 9

VI. FUZZY CONDITIONAL INFERENCE IN DECISION MAKING

Decision management usually happens in Decision Support Systems.

Example 1
Consider Business rule
If \( x \) is Demand of the product then \( x \) is High Price
Let \( x_1, x_2, x_3, x_4, x_5 \) be the Items.
The Generalized fuzzy set
\[
\mu_{\text{Demand}}^{\text{FCF}}(x) = 0.5/x_1 + 0.44/x_2 + 0.79/x_3 + 0.33/x_4 + 0.68/x_5
\]
\[
\mu_{\text{High Price}}^{\text{FCF}}(x) = 0.49/x_1 + 0.52/x_2 + 0.35/x_3 + 0.4/x_4 + 0.3/x_5,
\]
\[
\mu_{\text{High Price}}^{\text{FCF}}(x) = 0.49/x_1 + 0.52/x_2 + 0.35/x_3 + 0.4/x_4 + 0.3/x_5
\]

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Zadeh inference is given as \( A \rightarrow B = \min \{1, 1 - \mu_A(x) + \mu_B(x) \} \)

\[
\mu_{\text{Demand} \rightarrow \text{High Price}}^{\text{FCF}}(x) = 0.9/x_1 + 1/x_2 + 0.5/x_3 + 1/x_4 + 0.52/x_5 \\
\mu_{\text{Demand} \rightarrow \text{High Price}}^{\text{FCF}}(x) \geq 0.6 = 1/x_1 + 1/x_2 + 0/x_3 + 1/x_4 + 0/x_5
\]

Mamdani inference is given as \( A \rightarrow B = \min \{\mu_A(x) \cdot \mu_B(x) \} \)

\[
\mu_{\text{Demand} \rightarrow \text{High Price}}^{\text{FCF}}(x) = 0.4/x_1 + 0.44/x_2 + 0.29/x_3 + 0.33/x_4 + 0.2/x_5 \\
\mu_{\text{Demand} \rightarrow \text{High Price}}^{\text{FCF}}(x) \geq 0.6 = 1/x_1 + 1/x_2 + 0/x_3 + 1/x_4 + 0/x_5
\]

Mamdani inference is given as \( A \rightarrow B = \min \{\mu_A(x) \} \)

\[
\mu_{\text{Demand} \rightarrow \text{High Price}}^{\text{FCF}}(x) = 0.4/x_1 + 0.44/x_2 + 0.29/x_3 + 0.33/x_4 + 0.2/x_5 \\
\mu_{\text{Demand} \rightarrow \text{High Price}}^{\text{FCF}}(x) \geq 0.6 = 1/x_1 + 1/x_2 + 0/x_3 + 1/x_4 + 0/x_5
\]

**Example 2**

Consider Medical Diagnosis

If x has infection in the leg then surgery

Let \( x_1, x_2, x_3, x_4, x_5 \) are the Patients.

The fuzzy set

\[
\mu_{\text{Infection}}^{\text{FCF}}(x) = 0.76/x_1 + 0.78/x_2 + 0.46/x_3 + 0.86/x_4 + 0.58/x_5, \\
0.16/x_1 + 0.12/x_2 + 0.06/x_3 + 0.14/x_4 + 0.05/x_5 \\
= 0.6/x_1 + 0.64/x_2 + 0.4/x_3 + 0.72/x_4 + 0.53/x_5 \\
\mu_{\text{Surgery}}^{\text{FCF}}(x) = 0.59/x_1 + 0.26/x_2 + 0.55/x_3 + 0.24/x_4 + 0.35/x_5, \\
0.09/x_1 + 0.06/x_2 + 0.05/x_3 + 0.04/x_4 + 0.03/x_5 \\
= 0.5/x_1 + 0.2/x_2 + 0.5/x_3 + 0.2/x_4 + 0.32/x_5
\]

Using inference rule \( A \rightarrow B = \min \{1, 1 - \mu_A(x) + \mu_B(x) \} \)

\[
\mu_{\text{Infection} \rightarrow \text{Surgery}}^{\text{FCF}}(x) = 0.9/x_1 + 0.56/x_2 + 0.9/x_3 + 1/x_4 + 1/x_5 \\
\mu_{\text{Infection} \rightarrow \text{Surgery}}^{\text{FCF}}(x) = 1 \mu_{\text{Infection} \rightarrow \text{Surgery}}^{\text{FCF}}(x) \geq 6 \\
0 \mu_{\text{Infection} \rightarrow \text{Surgery}}^{\text{FCF}}(x) < 6
\]

The fuzzy risk set R is

\[
1/x_1 + 0/x_2 + 1/x_3 + 1/x_4 + 1/x_5
\]

The decision is to take surgery is Yes for \( x_1, x_3, x_4, x_5 \) and No for \( x_2 \).

The fuzzy reasoning under Risk management in Decision Support Systems may be Consider the fuzzy rule and fuzzy fact

If x has Infection of the product then x is to go for Surgery

x has very Infection

x is very Infection \( \rightarrow \) Surgery

\[
\mu_{\text{Infection} \rightarrow \text{Surgery}}^{\text{FCF}}(x) = 0.9/x_1 + 0.56/x_2 + 0.9/x_3 + 1/x_4 + 1/x_5 \\
\mu_{\text{very Infection}}^{\text{FCF}}(x) = 0.25/x_1 + 0.20/x_2 + 0.25/x_3 + 0.04/x_4 + 0.1/x_5
\]

x is very Demand \( \rightarrow \) Demand \( \rightarrow \) Increase Price

\[
= 0.35/x_1 + 0.66/x_2 + 0.35/x_3 + 0.04/x_4 + 0.1/x_5
\]

The decision is to take for surgery is Yes for \( x_2 \) and No for \( x_1, x_4, x_4 \) and \( x_5 \).

**VII. Conclusion**

The decision has to be taken under incomplete information in many applications like Business, Medicine etc. The fuzzy logic is used to deal with incomplete information. The fuzzy Decision set is defined with two-fold fuzzy set. The fuzzy logic is discussed with two-fold fuzzy set. The fuzzy Decision set, inference and reasoning are studied. The Business applications is discussed for fuzzy Decision set.
References Références Referencias


