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Modified MEWMA Control Scheme for an Analytical Process Data

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Strictly as per the compliance and regulations of:

Modified MEWMA Control Scheme for an Analytical Process Data

Alpaben K. Patel [α] & Jyoti Divecha [σ]

Abstract - This article introduces Multivariate Modified Exponentially Weighted Moving Average (MMOEWMA) control chart, a chart for detecting shifts of all kinds in case of highly first order vector autoregressive VAR (1) process . This chart is based on modified MEWMA control chart statistic which is a correction of MEWMA chart statistic. The performance of MMOEWMA chart is illustrated along with MEWMA chart for a chemical process data. The average run length (ARL) properties of MMOEWMA scheme are derived using Markov Chain approach. Algorithm for the ARL computation and R-program of monitoring MMOEWMA control chart are provided.

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I. Introduction

Multivariate statistical process control is often used in chemical and process industries where autocorrelation is most prevalent. Traditional multivariate statistical process control techniques are based on the assumption that the successive observation vectors are independent. In recent years, due to automation of measurement and data collection systems, a process can be sampled at higher rates, which ultimately leads to autocorrelation. Consequently, when the autocorrelation is present in the data, it can have a serious impact on the performance of classical control charts. This point has been made by numerous authors, including Berthouex, Hunter, and Pallensen (1978), Harris and Ross (1991), Montgomery and Mastrangelo (1991). Runger (1996) has presented a realistic model that generates autocorrelation and cross correlation and provides a useful approach to characterizing process data. The interpretation of these charts: charts based on modeling residuals is not as simple as the authors suggest, and the alternative engineering feedback control methods are often more appropriate with such highly auto correlated data.

This article considers the problem of monitoring the mean vector of a process in which observations can be a highly first order vector autoregressive VAR (1) and propose a control chart called Multivariate Modified EWMA chart. Multivariate Modified EWMA chart as a modification in MEWMA (Lowery et. al, 1992) chart statistic. Multivariate Modified EWMA chart that combines the features of multivariate Shewhart chart (Hotelling, 1947) and MEWMA chart in a simple way and has ability to detect small as well as large shift as soon as possible as required by some industrial processes with high level of first order vector autoregressive data.

MMOEWMA control statistic gives weight to the past observation vectors in slightly different way than MEWMA and each current change is considered with full weight. This corrects MEWMA statistic for suffering from inertia problem. This article discusses the procedures to construct the Multivariate Modified EWMA chart. Simulate the average run length to assess the performance of the chart. The MMOEWMA vector auto correlated control chart is defined in second section and the derivation of upper control limits is kept with Appendix 1. Further, performance of MMOEWMA monitoring scheme is illustrated along with MEWMA scheme for real multivariate chemical process data in third section. ARL properties of MMOEWMA are derived and compared with MEWMA in fourth section. The comparisons reveals that MMOEWMA scheme outperform MEWMA scheme. Computation of ARL values were carried out using Markov chain approach described in Appendix 2.

II. Multivariate Control Charts for Monitoring the Process Mean

Suppose that the **p** x **1** random vectors Y_1, Y_2, Y_3, … each representing the p quality characteristics to be monitored, are observed over time. These vectors may represent individual observations or sample mean vectors. To study the performance of the various multivariate control charts, it will be assumed that $\mathbf{Y}_n$, n = 1, 2, …, are independent multivariate normal random vectors with mean vectors $\boldsymbol{\mu}_n$, n =1, 2,… For simplicity, it is assumed that each of the random vectors has the known co-variance matrix $\boldsymbol{\Sigma}$.

a) The Multivariate Shewhart Control Chart

Hotelling's (1947) introduced a multivariate control-chart procedure based on his Hotelling's chi-square statistics defined as $\chi_n^2 = Y_n^{'}\Sigma^{-1}Y_n$. At any n[th] stage in process,

$$\chi_n^2 = Y_n^{'}\Sigma^{-1}Y_n > h, \tag{1}$$

signals that a statistically significant shift in the mean has occurred; that is process out-of-control, where *h* >

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0 is a specified control limit. Because this procedure is based on only the most recent observation, it is insensitive to small and moderate shifts in the mean vector.

b) The Multivariate EWMA (MEWMA) control chart

Lowery et al. (1992) proposed the MEWMA chart as natural extension of EWMA chart. It is a popular chart used to monitor a process with p quality characteristics for detecting small to moderate shifts. The in-control process mean is assumed without loss of generality to be a vector of zeros, and covariance matrix Σ. The MEWMA control statistic is defined as vectors,

$$X_n = \Lambda Y_n + (I-\Lambda)X_{n-1},\ n=1,2,\ldots, \tag{2}$$

where $X_0 = \mathbf{0}$, 1 x p vector and $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2,\ldots, \lambda_p)$, $0<\lambda j\leq 1$, $j=1,2,\ldots,p$.

The MEWMA chart gives an out-of-control signal as soon as

$$T_{n1}^2 = X_n' \Sigma_{xn}^{-1} X_n > h_1, \tag{3}$$

where h_1 (>0) is chosen to achieve a specified in control ARL and Σ_{xn} is the covariance matrix of X_n given by $\Sigma_{xn} = \{\lambda/(2-\lambda)\}\Sigma$, under equality of weights of past observations for all p characteristics; $\lambda_1 = \lambda_2 = \ldots = \lambda_p = \lambda$.

The UCL $= \left(\dfrac{\lambda}{2-\lambda}\right)^{1/2} (h_1)^{1/2}$. If one or more points fall beyond h_1, the process is assumed to be out-of-control. The magnitude of the shift is reflected in the non-centrality parameter $\boldsymbol{\mu}_1' \Sigma^{-1} \boldsymbol{\mu}_1$. They conclude that an assignable causes result in a shift in the process mean from $\boldsymbol{\mu}_0$ to $\boldsymbol{\mu}_1$.

c) MMOEWMA control chart for monitoring the first order vector autoregressive VAR (1) process mean

The MMOEWMA chart as natural extension of Patel and Divecha (2011) proposed Modified EWMA chart. The desirable properties of a multivariate auto correlated control chart are that it is easy to implement and is effective for detecting shifts of all sizes as per technical specifications. The Multivariate Modified EWMA chart that introduce considers past observations similar to MEWMA scheme and additionally considers past as well as latest change in the process. Let Y_n, $n = 1,2,\ldots,$ are sequence of first order auto correlated normal random vectors with mean vector $\boldsymbol{\mu}_n$, and common covariance matrix Σ. Further it is assumed without loss of generality that the in control process mean vector is $\boldsymbol{\mu}_0 = (0, 0,\ldots, 0)' = \mathbf{0}$.

The MMOEWMA chart statistic is a modification in MEWMA chart statistic. To define MMOEWMA control statistic as vector X_n given by,

$$X_n = \Lambda Y_n + (I-\Lambda)X_{n-1} + (Y_n - Y_{n-1}),\ n\geq 1, \tag{4}$$

where X_0 is the p-dimensional zero vector and $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2,\ldots, \lambda_p)$, $0<\lambda j\leq 1$, $j=1,2,\ldots,p$. The MMOEWMA chart gives an out-of-control signal as soon as

$$T_{n2}^2 = X_n' \Sigma_{xn}^{-1} X_n > h_2, \tag{5}$$

where h_2 (>0) is chosen to achieve a specified in control ARL. If one or more points fall beyond h_2, the process is assumed to be out-of-control. Σ_{xn} is the covariance matrix of X_n given by $\Sigma_{xn} = \left(\dfrac{\lambda}{2-\lambda} + \dfrac{2\lambda(1-\lambda)}{2-\lambda}\right)\Sigma$, under equality of weights of past observations for all p characteristics; $\lambda_1 = \lambda_2 = \ldots = \lambda_p = \lambda$, and past and current changes. The upper control limit of MMOEWMA chart is,

UCL $= \left(\dfrac{\lambda}{2-\lambda} + \dfrac{2\lambda(1-\lambda)}{2-\lambda}\right)^{1/2} (h_2)^{1/2}$ (discussed in Appendix 1).

III. Illustration

Table 1 : Temperatures Data

NO	Observations			Deviated Observations		
	Y_1	Y_2	Y_3	$Y_1-\mu_{01}$	$Y_2-\mu_{02}$	$Y_3-\mu_{03}$
0	92.24	95.56	100.27	0.00	0.00	0.00
1	92.83	95.16	100.77	0.59	-0.40	0.50
…	…	…	…	…	…	…
12	92.84	95.16	100.76	0.60	-0.40	0.49
13	92.84	95.16	100.76	0.60	-0.40	0.49
14	92.84	95.16	100.76	0.60	-0.40	0.49
15	92.84	95.16	100.76	0.60	-0.40	0.49
…	…	…	…	…	…	…

279	92.59	95.10	100.47	0.35	-0.46	0.20
280	92.59	95.10	100.46	0.35	-0.46	0.19
281	92.58	95.10	100.46	0.34	-0.46	0.19
282	92.58	95.10	100.46	0.34	-0.46	0.19
…	…	…	…	…	…	…
588	91.37	95.27	99.73	-0.87	-0.27	-0.54
589	91.37	95.27	99.72	-0.88	-0.29	-0.55
590	91.36	95.27	99.72	-0.88	-0.29	-0.55
591	91.36	95.27	99.72	-0.88	-0.29	-0.55
592	91.35	95.28	99.72	-0.89	-0.28	-0.55
…	…	…	…	…	…	…
998	92.30	96.22	100.16	0.06	0.66	-0.11
999	92.30	96.21	100.16	0.06	0.65	-0.11
…	…	…	…	…	…	…
1200	92.67	95.84	100.72	0.43	0.28	0.45
1201	92.67	95.83	101.95	0.43	0.27	1.68
1202	92.67	95.83	100.73	0.43	0.27	0.46
…	…	…	…	…	…	…
1416	92.55	95.35	100.90	0.31	-0.21	0.63
1417	92.55	95.35	100.90	0.31	-0.21	0.63
…	…	…	…	…	…	…
1434	92.54	95.31	100.87	0.30	-0.26	0.60
1435	92.53	95.30	100.87	0.29	-0.26	0.60
…	…	…	…	…	…	…
1439	92.53	95.29	100.86	0.29	-0.27	0.59

a) MMOEWMA chart for monitoring Multivariate Chemical Process using table 1 and table 2

Table 1 displays the part of measurements on three temperature column taken every minute from a chemical process that is working in control and out of control situations, abrupt changes and small shifts occur. Here number of variable p=3. Three variables are temperature columns Y_1 with mean μ_{01}=92.24, Y_2 with mean μ_{02}=95.56, Y_3 with mean μ_{03}= 100.27. To assume covariance matrix to be

$$\sum = \begin{pmatrix} V(Y_{11}) & C(Y_1,Y_2) & C(Y_1,Y_3) \\ C(Y_2,Y_1) & V(Y_{22}) & C(Y_2,Y_3) \\ C(Y_3,Y_1) & C(Y_3,Y_2) & V(Y_{33}) \end{pmatrix} = \begin{pmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0.5 \\ 0.5 & 0.5 & 1 \end{pmatrix}$$

Upper control limit of MEWMA UCL $= \left(\frac{\lambda}{2-\lambda}\right)^{1/2} (h_1)^{1/2}$ =0.882, where λ=0.1, h_1=14.7, and MMOEWMA UCL $= \left(\frac{\lambda}{2-\lambda} + \frac{2\lambda\,(1-\lambda)}{2-\lambda}\right)^{1/2} (h_2)^{1/2}$ =0.839, where λ=0.1, h_2 = 5.417. UCL is used in average run length to choose appropriate value of decision interval. The MMOEWMA chart gives an out-of-control signal as soon as

$$T_{n2}^2 = X_n^{'}\,\boldsymbol{\Sigma}_{xn}^{-1}\,X_n > h_2, \qquad h_2 = 5.417$$

and the MEWMA chart gives an out-of-control signal as soon as

$$T_{n1}^2 = X_n^{'}\,\boldsymbol{\Sigma}_{xn}^{-1}\,X_n > h_1,\ h_1 = 14.78.$$

Table 2 shows that, MMOEWMA vector gives best forecasts for the process mean vector; undoubtedly better than the MEWMA prediction barring abrupt change situation (1201st observation). MMOEWMA also detects all the shifts more timely as compared to MEWMA for chemical process data.

Table 2 : Monitoring performance of MEWMA and MMOEWMA for the Chemical Process three variate temperature Data

No.	MEWMA vector, λ=0.1, h_1 = 14.78			MEWMA statistics	MMOEWMA vector, λ=0.1, h_2 = 5.417			MMOEWMA statistics
	X_1	X_2	X_3	T_{n1}^2	X_1	X_2	X_3	T_{n2}^2
1	0.06	-0.04	0.05	0.24	0.56	0.46	0.55	2.91
…	…	…	…	…	…	…	…	…
12	0.43	-0.29	0.36	12.48	0.58	-0.14	0.51	5.19
13	0.44	-0.30	0.37	13.48	0.58	-0.16	0.50	**5.47***
14	0.47	-0.31	0.38	14.43	0.58	-0.19	0.50	5.73*
15	0.47	-0.32	0.39	**15.30***	0.58	-0.21	0.50	5.98*
…	…	…	…	…	…	…	…	…
279	0.39	-0.46	0.21	15.33*	0.36	-0.48	0.19	5.46*
280	0.38	-0.46	0.21	15.17*	0.36	-0.48	0.19	5.40
281	0.38	-0.46	0.21	15.02*	0.36	-0.48	0.19	5.35
282	0.37	-0.46	0.21	14.87*	0.35	-0.48	0.19	5.30
…	…	…	…	…	…	…	…	…
588	-0.84	-0.29	-0.52	14.26	-0.86	-0.32	-0.55	5.37
589	-0.84	-0.29	-0.52	14.41	-0.86	-0.32	-0.55	**5.42***
590	-0.84	-0.29	-0.53	14.56	-0.87	-0.31	-0.55	5.47*
591	-0.85	-0.29	-0.53	14.72	-0.87	-0.31	-0.55	5.53*
592	-0.85	-0.29	-0.53	**14.87***	-0.87	-0.31	-0.56	5.59*
…	…	…	…	…	…	…	…	…
998	0.02	0.67	-0.13	14.91*	0.05	0.70	-0.11	5.44*
999	0.03	0.67	-0.13	14.75	0.06	0.70	-0.10	5.38
…	…	…	…	…	…	…	…	…
1200	0.43	0.29	0.43	4.72	0.46	0.32	0.46	1.91
1201	0.43	0.29	0.55	6.49	**1.68**	**1.54**	**1.81**	**29.10***
1202	0.43	0.29	0.54	6.33	0.34	0.19	0.45	1.54
…	…	…	…	…	…	…	…	…
1416	0.31	-0.18	0.64	14.73	0.31	-0.19	0.63	5.24
1417	0.31	-0.19	0.63	**14.79***	0.30	-0.19	0.63	5.25
…	…	…	…	…	…	…	…	…
1434	0.30	-0.23	0.61	15.35*	0.28	-0.25	0.60	5.41
1435	0.30	-0.23	0.61	15.38*	0.28	-0.25	0.60	**5.43***
…	…	…	…	…	…	…	…	…
1439	0.30	-0.24	0.61	15.48*	0.28	-0.26	0.59	5.46*

In Table 2 observe that, MEWMA control chart detects shifts on observation 15th to 282nd, 592nd to 998th, and 1417th to 1439th. The MMOEWMA control chart detects shifts on observation 13th to 279th, 589th to 998th, and 1435th to 1439th. MMOEWMA chart detect abrupt change at observation 1201st, but MEWMA could not detect it.

Figure 1 and 2 depict MEWMA and MMOEWMA statistics charting.

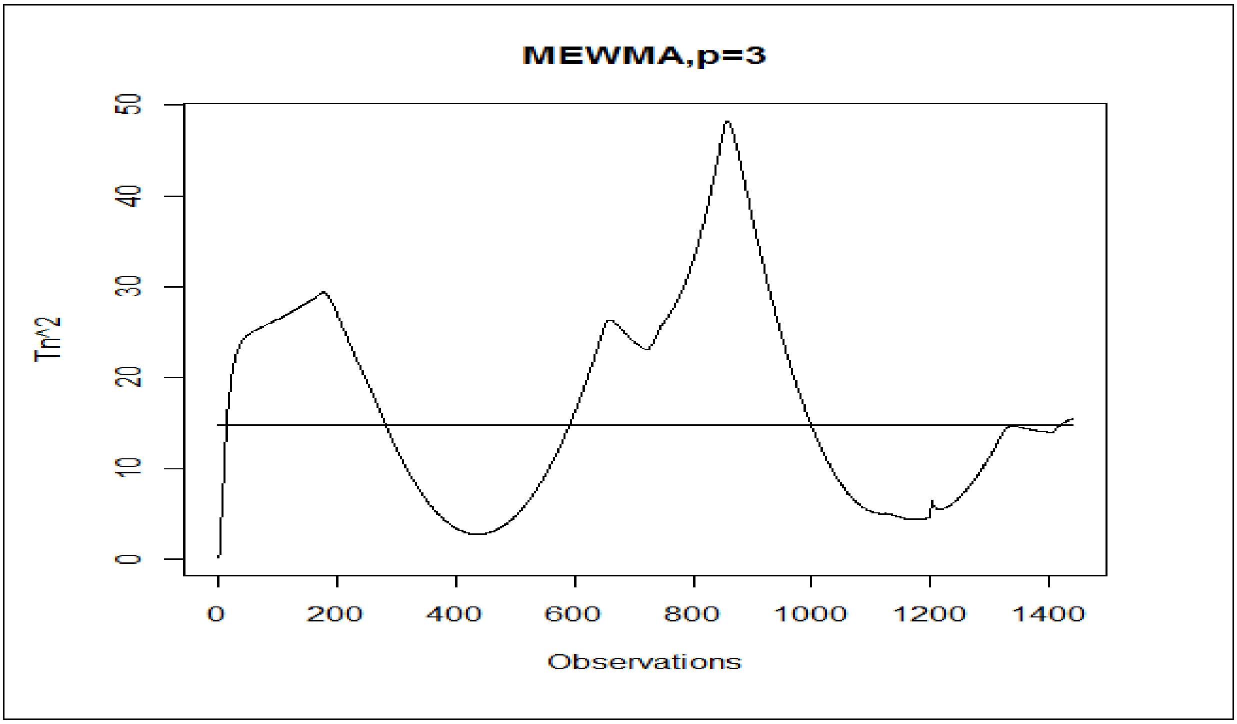

Figure 1 : MEWWA Control Chart (p=3)

Observe the shoot up bar showing abrupt shift in figure 2 which is completely missing in figure 1.

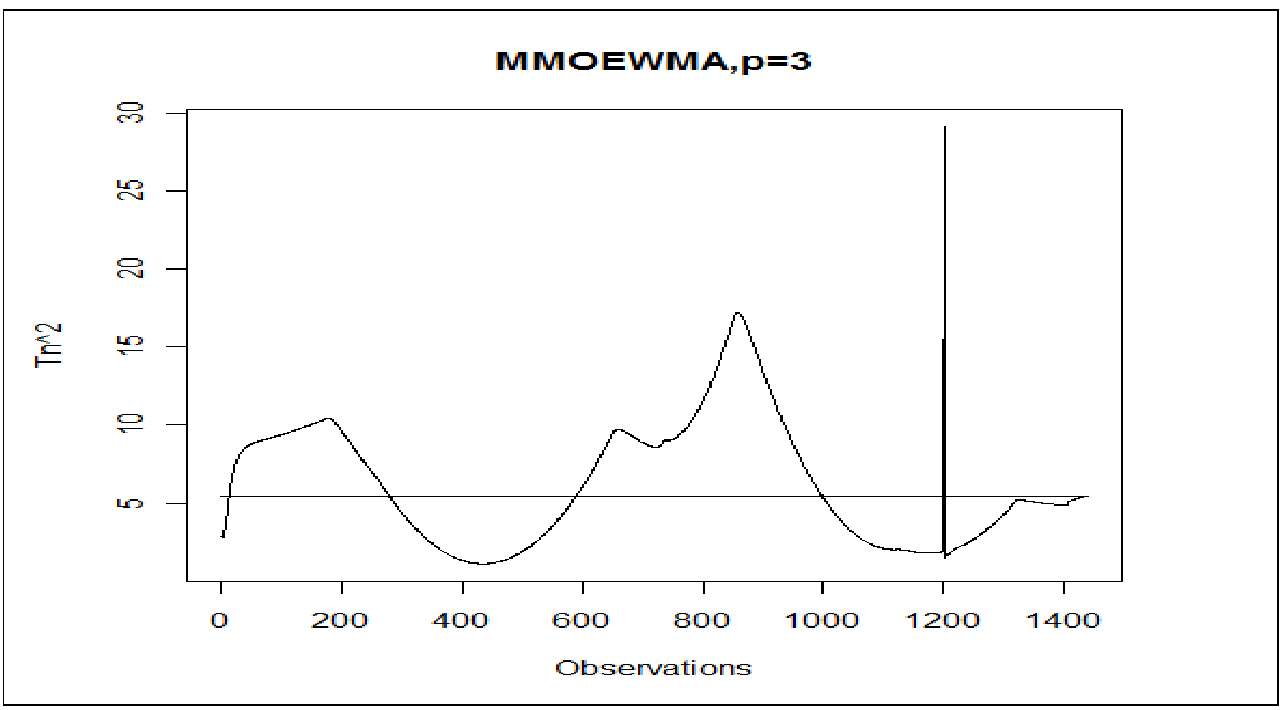

Figure 2 : MMOEWMA Control Chart (p=3)

Note that 1201st run has abrupt shift in variable Y_3.

b) Properties of MMOEWMA scheme and comparison with MEWMA scheme

All the ARL computations were carried out using Markov chain approach described in Appendix 2. MMOEWMA is the chart for multivariate processes having autocorrelated observations. However, assuming that MEWMA chart can be applied at least to the residual vectors, to compare the ARL values of MMOEWMA with that of MEWMA having common parameters.

Table 3 : Average Run Lengths of MEWMA Charts

ARL Values for MEWMA Charts (p=2, p=3 and p=4) from Lowery et. al. (1992)						
h_1=	10.75	12.34	13.10	14.78	15.16	16.94
Shift	P =2 and λ =0.10		p =3 and λ = 0.1		p =4 and λ = 0.1	
0.0	501	999	502	1007	497	995
0.5	39.5	51.1	45.6	61.2	52.3	68
1.0	12.1	13.7	13.5	15.3	14.5	16.5
1.5	7.03	7.69	7.66	8.40	8.20	8.99
2.0	4.97	5.39	5.43	5.83	5.79	6.23
2.5	3.90	4.23	4.26	4.54	4.51	4.81
3.0	3.27	3.49	3.52	3.75	3.74	3.97

Table 4 : Average Run Lengths of MMOEWMA Charts

ARL Values for Multivariate Modified EWMA (MMOEWMA) Charts (p=2, p=3 and p=4)						
h_2=	3.93	4.525	4.78	5.417	5.546	6.221
Shift	p =2 and λ =0.1		p =3 and λ = 0.1		p =4 and λ = 0.1	
0.0	500	1000	500	1000	500	1000
0.5	30.5	40.09	31.52	41.57	32.3	42.77
1.0	10.06	11.63	10.54	12.15	10.89	12.55
1.5	5.69	6.38	6.04	6.73	6.30	7.0
2.0	3.87	4.29	4.16	4.58	4.38	4.80
2.5	2.87	3.17	3.12	3.42	3.31	3.61
3.0	2.02	2.24	2.25	2.46	2.62	2.63

Table 3 to table 6 showed that the control limits for MMOEWMA are quite small than those of MEWMA as well as χ^2 chart for the same in control ARL. For two, three, four variable cases, the smaller out of control ARL imply that MMOEWMA chart performs excellent in detection of shifts, be it small, moderate or large for every of in control ARLs. MMOEWMA chart with $\lambda_1 = \lambda_2 = \ldots = \lambda_p = \lambda = 1$ is multivariate Shewhart chart version for multivariate autocorrelated process.

Table 5 : Average Run Lengths of Multivariate Charts

	χ^2 chart	MEWMA Chart				
λ =		0.1	0.2	0.4	0.6	0.8
Shift	*h*	h_1				
	10.6	8.66	9.65	10.29	10.53	10.58
	ARL values for p=2 from Lowery et. al. (1992)					
0	200.	200	201	199	200	200
0.5	116.	28.1	35.10	51.9	73.6	95.5
1.0	42.	10.2	10.10	13.2	19.3	28.1
1.5	15.8	6.12	5.50	5.74	7.24	10.3
2.0	6.9	4.41	3.80	3.54	3.86	4.75
2.5	3.5	3.51	2.91	2.55	2.53	2.75
3.0	2.2	2.92	2.42	2.04	1.88	1.91

Table 6 : Average Run Lengths of MMOEWMA Charts

	MMOEWMA Chart					
λ =	0.1	0.2	0.3	0.4	0.6	0.8
h_2=	3.135	3.742	4.223	4.705	5.85	7.561
Shift	ARL values for p=2					
0	200	200	200	200	200	200
0.5	21.1	24.76	29.9	29.9	51.6	74.98
1.0	8.12	7.88	8.65	8.65	14.93	23.99
1.5	4.78	4.11	4.06	4.06	5.75	8.92
2.0	3.29	2.64	2.44	2.44	2.92	4.12
2.5	2.45	1.87	1.69	1.69	1.86	2.36
3.0	1.90	1.43	1.32	1.32	1.41	1.63

IV. Conclusion

A simple multivariate control chart for monitoring small as well as large shifts in highly first order vector autoregressive VAR (1) process such as multivariate chemical process is given. It is good method to monitor first order vector autoregressive process in chemical/other industries.

Appendix 1 Derivation of the Covariance Matrix of MMOEWMA Statistic X_N

By repeated substitution in equation $X_n = \mathbf{\Lambda} Y_n + (I - \mathbf{\Lambda})X_{n-1} + (Y_n - Y_{n-1})$, $n \geq 1$, it can be shown that

$$X_n = +(I-\Lambda)^n Y_0 + \sum_{j=0}^{n-1}(I-\Lambda)^j (Y_{n-j} - Y_{n-j-1}) \quad \text{(i)}$$

The expectation of X_n gives, $E(X_n) = \boldsymbol{\mu}$, (mean of Y_n).

Lemma 1: If $\lambda_1 = \lambda_2 = \ldots = \lambda_p = \lambda$, then the expression for Multivariate Modified EWMA statistic

$$X_n = \lambda \sum_{j=0}^{n-1} (1-\lambda)^j Y_{n-j} + (1-\lambda)^n Y_0 + \sum_{j=0}^{n-1} (1-\lambda)^j (Y_{n-j} - Y_{n-j-1})$$

Taking expectation on both side,

$$E(X_n) = \lambda \sum_{j=0}^{n-1} (1-\lambda)^j E(Y_{n-j}) + (1-\lambda)^n E(Y_0) + \sum_{j=0}^{n-1} (1-\lambda)^j E(Y_{n-j} - Y_{n-j-1})$$

But $\lambda \sum_{j=0}^{n} (1-\lambda)^j = \dfrac{\lambda\,[1-(1-\lambda)^n]}{[1-(1-\lambda)]} = [1-(1-\lambda)^n]$

$\therefore$ $E(X_n) = [1-(1-\lambda)^n]\mu + (1-\lambda)^n \mu + 0$

$= \boldsymbol{\mu}$

Lemma 2 : If the starting value of process is, $X_0 = \mu_0 = Y_0$ and $0 < \lambda \leq 1$ is a constant. The mean is, $E(X_n) = E((1-\lambda)X_{n-1} + \lambda Y_n + (Y_n - Y_{n-1})) = \mu_0$.

For p = 2 and n=1, we have,

$$X_1 = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} Y_{11} \\ Y_{21} \end{bmatrix} + \begin{bmatrix} 1-\lambda_1 & 0 \\ 0 & 1-\lambda_2 \end{bmatrix} \begin{bmatrix} Y_{10} \\ Y_{20} \end{bmatrix} + \begin{bmatrix} Y_{11} - Y_{10} \\ Y_{21} - Y_{20} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda_1 Y_{11} + (1-\lambda_1)Y_{10} + Y_{11} - Y_{10} \\ \lambda_2 Y_{21} + (1-\lambda_2)Y_{20} + Y_{21} - Y_{20} \end{bmatrix} = \begin{bmatrix} X_{11} \\ X_{21} \end{bmatrix} \text{ say,}$$

So that,

$$\mathrm{Cov}(X_1) = \sum\nolimits_{X1} = \begin{bmatrix} V(X_{11}) & Cov(X_{11}, X_{21}) \\ & V(X_{21}) \end{bmatrix} \quad \text{(ii)}$$

Where $V(X_{11})$ and $V(X_{21})$ are the variance of univariate modified EWMA statistic, and $Cov(X_{11}, X_{21}) = \lambda_1 \lambda_2 \sigma_{12} + (1-\lambda_1)(1-\lambda_2)\sigma_{12} + \mathrm{cov}(Y_{11} - Y_{10}, Y_{21} - Y_{20})$.

Then as per (ii)

$$\mathrm{Cov}(X_n) = \sum\nolimits_{Xn} = \begin{bmatrix} V(X_{1n}) & Cov(X_{1n}, X_{2n}) \\ & V(X_{2n}) \end{bmatrix}$$

Now Y_n's are autocorrelated normal with covariance matrix $\boldsymbol{\Sigma}$, so that the $(Y_n - Y_{n-1})$'s ($n \geq 1$) have covariance matrix 2(**I-Rho**)$\boldsymbol{\Sigma}$ with **Rho** = diag($\rho_{y1}, \rho_{y2}, \ldots, \rho_{yp}$). Then, taking $\lambda_1 = \lambda_2 = \ldots = \lambda_p = \lambda$ and $\rho_{y1}, \rho_{y2}, \ldots, \rho_{yp} \to 1$, as n tends to infinity.

Lemma 3: The variance of univariate Modified EWMA (MOEWMA) control statistic X_n is,

$$V(X_n) = V(\lambda \sum_{j=0}^{n-1} (1-\lambda)^j Y_{n-j}) + V((1-\lambda)^n Y_0) + V(\sum_{j=0}^{n-1} (1-\lambda)^j (Y_{n-j} - Y_{n-j-1}))$$

$$V(X_n) = (1-\lambda)^{2n} V(Y_0) + \sum_{j=0}^{n-1} \lambda^2 (1-\lambda)^{2j} V(Y_{n-j}) + 2\sum_{j=0}^{n-1} \lambda^2 (1-\lambda)^{2j+1} Cov(Y_{n-j}, Y_{n-j-1}) +$$

$$\sum_{j=0}^{n-1} (1-\lambda)^{2j} V(Y_{n-j} - Y_{n-j-1}) + 2\sum_{j=0}^{n-1} (1-\lambda)^{2j+1} Cov[(Y_{n-j} - Y_{n-j-1}), (Y_{n-j-1} - Y_{n-j-2})]$$

$$+\sum_{j=0}^{n-1} \lambda (1-\lambda)^{2j} Cov\big(Y_{n-j}, (Y_{n-j} - Y_{n-j-1})\big) + \sum_{j=0}^{n-1} \lambda (1-\lambda)^{2j+1} Cov\big(Y_{n-j-1}, (Y_{n-j} - Y_{n-j-1})\big)$$

Since Y_n's are autocorrelated normal with variance σ^2, the variance of (Y_n-Y_{n-1}) ($n\geq1$) is $\sigma_1^2 = 2\sigma^2 - 2\rho\sigma^2 = 2(1-\rho)\sigma^2$ (small when $\rho\to1$). The weights $\lambda(1-\lambda)^{2j}$ decrease geometrically with the age of sample mean. Suppose Y_n's are correlated to the forward fluctuation (Y_n-Y_{n-1}) ($n \geq 1$) with common correlation ρ_1 and correlated to the backward fluctuation (Y_{n+1}-Y_n), ($n\geq0$) with common correlation ρ_2, and forward fluctuation (Y_n-Y_{n-1}) are correlated to the backward fluctuation (Y_{n+1}-Y_n), ($n\geq1$) with common correlation ρ_3, then asymptotic variance for large n,

$$V(X_n) = \frac{\lambda}{(2-\lambda)}\sigma^2 + \frac{2\lambda(1-\lambda)}{(2-\lambda)}\rho\sigma^2 + \frac{2(1-\rho)\sigma^2}{\lambda(2-\lambda)} + \frac{4\rho_3(1-\rho)(1-\lambda)\sigma^2}{\lambda(2-\lambda)} + \frac{2\sqrt{2}\rho_1\sqrt{(1-\rho)}\sigma^2}{(2-\lambda)} + \frac{(1-\lambda)2\sqrt{2}\rho_2\sqrt{1-\rho}\sigma^2}{(2-\lambda)} \qquad \text{(iii)}$$

In normal autocorrelated process (a) with ρ_3 nearly negative half and ρ_1, ρ_2 nearly equal and opposite in sign and being monitored for small shifts, (b) with autocorrelation ρ nearly one ($\rho\to1$) the above expression (iii), reduces to

$$V(X_n) = \frac{\lambda}{(2-\lambda)}\sigma^2 + \frac{2\lambda(1-\lambda)}{(2-\lambda)}\rho\sigma^2 \qquad \text{(iv)}$$

Let $\frac{2\lambda(1-\lambda)}{(2-\lambda)}\rho\sigma^2$ is a small value for high value of ρ and small λ, sometimes even negligibly small such that modified EWMA limits equal EWMA limits.

Therefore, $V(X_n) = \left[\frac{\lambda}{(2-\lambda)} + \frac{2\lambda(1-\lambda)}{(2-\lambda)} \right] \sigma^2$. (v)

Therefore, Multivariate MOEWMA covariance from equation (iv, v) becomes

$$V(X_{1n}) = \left[\frac{\lambda}{(2-\lambda)} + \frac{2\lambda(1-\lambda)}{(2-\lambda)} \right] \Sigma = V(X_{2n}) = Cov(X_{1n}, X_{2n}).$$

In general the best approximation of covariance matrix of the MMOEWMA p-variable vectors is given by,

$$\sum_{Xn} = \frac{\lambda}{(2-\lambda)}\Sigma + \frac{2\lambda(1-\lambda)}{(2-\lambda)}\rho\Sigma \qquad \text{(vi)}$$

The UCL of MMOEWMA control chart is

$$\text{UCL} = \left(\frac{\lambda}{(2-\lambda)} + \frac{2\lambda(1-\lambda)}{(2-\lambda)} \right)^{1/2} (h_2)^{1/2} \qquad \text{(vii)}$$

The MMOEWMA chart gives an out-of-control signal as soon as

$$T_{n2}^2 = X_n' \boldsymbol{\Sigma}_{xn}^{-1} X_n > h_2, \qquad \text{(viii)}$$

Where h_2 (>0) is chosen to achieve a specified in control ARL. If one or more points fall beyond h_2, the process is assumed to be out-of-control. The magnitude of the shift is reflected in the non-centrality parameter $\boldsymbol{\mu}_1'\Sigma^{-1}\boldsymbol{\mu}_1$. We conclude that an assignable causes result in a shift in the process mean from $\boldsymbol{\mu}_0$ to $\boldsymbol{\mu}_1$.

Appendix 2 ARL Computation for MMOEWMA Scheme using Markov Chain Approach

Following Runger and Prabhu (1996) and Molnau et al. (2001) the Markov chain approach of ARL for MMOEWMA has been derived. Different choices of λ (weighting factor), h_2 (decision value), and p (number of variable) are considered.

In and out of-Control Case

For the in or out control case, the ARL analysis can be simplified as a one dimensional Markov chain.

To approximate $\|X_n\|$, we partition the control region into m+1 transient states, each of width g = $\frac{2UCL}{(2m+1)}$.

In this case the two dimensional range of X_n is represented by the X_1 and X_2 axes, and the states used for the Markov chain are assumed as circular rings. Because Y_n has a spherical distribution, the probability of transitioning from state i to state j, denoted as p(i, j), depends only on the radii of states i and j. For i = 0,1,2,…,m and j not equal to zero.

p(i,j) = P (d_n in state j | d_{n-1} in state i)

= P [(j-0.5) g < $\|\lambda Y_n + (1-\lambda) X_{n-1} + (Y_n - Y_{n-1})\|$ < (j+0.5) g | d_{n-1} = ig].

Given that d_{n-1} = ig, $X_{n-1}$is distributed as igU, and j =0,1,2,…,m

p(i,j) = P [(j-0.5) g < $\|\lambda Y_n + (1-\lambda) igU + (Y_n - Y_{n-1})\|$ < (j+0.5) g | d_{n-1} = ig].

Let **e** denote the p component unit vector **e**' =(1,0,0,…,0). According to Runger and Prabhu (1996) Y_n and U are independent spherical random variables, without loss of generality it can assume that U is identity equal to e to obtain

p(i,j) = P [{(j-0.5) g }/ λ< $\|Y_n + [(1-\lambda) ige + (Y_n - Y_{n-1})]/\lambda\|$ < {(j+0.5) g} / λ].

Let $\chi^2(p,c)$ denote a non central chi square random variable with p degrees of freedom and non centrality parameter c. Then we have

For j not equal to zero (j ≠ 0),

$$p(i,j) = P\left[\frac{(j-0.5)^2 g^2}{\lambda^2} \prec \chi^2(p,c) \prec \frac{(j+0.5)^2 g^2}{\lambda^2}\right],$$

Where c = [{(1-λ) i g / λ}+d]^2, degree of freedom is p , d is the shift in mean vector.

For the case where j =0, we have

p(i,0) = P [χ^2 (p,c) < {$(0.5)^2$ g^2 / λ^2 }].

For any control chart that is approximated by a Markov chain, the run length performance can be determined from the transition probability matrix. Assume that a Markov chain has s states (see Brook and Evans (1972)). The transition probability matrix contains the transition probabilities for moving from state to state. Let this s x s matrix of transition probabilities be presented as **P**, where the process mean vector is such that the non centrality parameter is δ. Let the s x 1 vector **q** designate the starting state of the Markov chain. The vector **q** will have a one in the component corresponding to the starting state and zeros in all of the other components. The zero state ARL of a scheme modeled as a Markov chain represented by ARL=**q**'(**I-P**)$^{-1}$ **1**. (ix)

Steps of ARL Computation for MMOEWMA

Step-1 Choose the parameter λ (Weighting factor), h_2 (decision value), p (number of variable), and shift in mean vector d.

Step-2 The upper control limit of MMOEWMA chart is, UCL = $\left(\frac{\lambda}{2-\lambda} + \frac{2\lambda(1-\lambda)}{2-\lambda}\right)^{1/2} (h_2)^{1/2}$.

Step-3 Choose the number of states m.

Step-4 Compute width g = $\frac{2UCL}{(2m+1)}$.

Step-5 $\chi^2(p,c)$ denotes a non central chi square random variable with p degrees of freedom and non centrality parameter c.

Step-6 Non centrality parameter, c = [{(1-λ) i g / λ}+d]^2, degree of freedom is p , d is the shift in mean vector.

Step-7 For j not equal to zero (j ≠ 0),

$$p(i,j) = P\left[\frac{(j-0.5)^2 g^2}{\lambda^2} \prec \chi^2(p,c) \prec \frac{(j+0.5)^2 g^2}{\lambda^2}\right].$$

Step-8 For the case where j =0, we have

p(i,0) = P [χ^2 (p,c) < {$(0.5)^2$ g^2 / λ^2 }].

Step-9 Adjust the t. p.m.(R_a) such that row sums are unity.

Step-10 Compute u = [I - R]$^{-1}$ **1**

Step-11 Compute q = R_a'* I

Step-12 ARL = q' * u, OR ARL = q'[I - R]$^{-1}$ **1**

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R-Program for monitoring Modified Multivariate Exponentially Weighted Moving Average Control Chart

```
## Multivariate MOEWMA (MMOEWMA)
## Three Temperature Data
p<-3
X<-read.table("Rprogram/Temp3.txt",header=TRUE)
X
##Temperature T3=X1,T11=X2,T21=X3
X1<-as.matrix(X[1:1439,1])
X1
x1<-ts(X1)
ar1<-arima(x1,order=c(1,0,1))
a1<-mean(X1)
a1<-92.24
##a1=92.24
X2<-as.matrix(X[1:1439,2])
X2
x2<-ts(X2)
ar2<-arima(x2,order=c(1,0,1))
a2<-mean(X2)
a2<-95.56
## a2= 95.56
X3<-as.matrix(X[1:1439,3])
X3
x3<-ts(X3)
ar3<-arima(x3,order=c(1,0,1))
a3<-mean(X3)
a3<-100.27
## a3=100.27
## if we take unit variances and Correlation=0.5
cmat <- matrix(c(1,0.5,0.5,0.5,1,0.5,0.5,0.5,1), nrow = 3, ncol=3, byrow=TRUE)
cmat
cmat1<-solve(cmat)
cmat1
m<-1439
## Exponential Weight r, 0<r<=1
r<-0.10
```

```
## Difference Variance
##k<-0.095
k<-(2*r*(1-r))/(2-r)
Xn<-matrix(0,m,3)
for(i in 1:m)
{ for(j in 1:3)
{
Xn[i,1]<-X[i,1]-a1
Xn[i,2]<-X[i,2]-a2
Xn[i,3]<-X[i,3]-a3
}}
round(Xn,2)
## MMOEWMA Vector Zi= rXi+(1-r)Zi-1+(Xi-Xi-1), Z0=M0=X0=0
##Asymptotic Sz={(r/(2-r))+k}s
Sz<-{(r/(2-r))+k}*cmat
Si<-solve(Sz)
Si
Zi<-matrix(0,m,3)
Z1<-0
Z2<-0
Z3<-0
X0<-0
for(i in 1:m)
{  for(j in 1:3)
{
 Zi[i,1]<-r%*%Xn[i,1]+(1-r)*Z1+(Xn[i,j]-X0)
 Zi[i,2]<-r%*%Xn[i,2]+(1-r)*Z2+(Xn[i,j]-X0)
 Zi[i,3]<-r%*%Xn[i,3]+(1-r)*Z3+(Xn[i,j]-X0)
 if(i>1)
{
Zi[i,1]<-r%*%Xn[i,1]+(1-r)%*%Zi[i-1,1]+(Xn[i,j]-Xn[i-1,j])
Zi[i,2]<-r%*%Xn[i,2]+(1-r)%*%Zi[i-1,2]+(Xn[i,j]-Xn[i-1,j])
Zi[i,3]<-r%*%Xn[i,3]+(1-r)%*%Zi[i-1,3]+(Xn[i,j]-Xn[i-1,j])
}   }     }
round(Zi,2)
Tn<-matrix(0,m,1)
t1<-matrix(0,1,3)
s1<-matrix(0,3,3)
for(i in 1:m)
{  Tn[i,]<-t(Zi[i,])%*%Si[,]%*%Zi[i,] }
round(Tn,2)
h4<-matrix(5.417,m,1)
shift<-matrix(0,m,1)
for(i in 1:m)
{  if(Tn[i]>h4[i])
shift[i]<-1
else
shift[i]<-0
}
shift
n1<-matrix(0,m,1)
for(i in 1:m)
{ n1[i]<-i}
n1
plot(n1,Tn,type="l",xlab="Observations",ylab="Tn^2",main="MMOEWMA,p=3")
lines(h4)
##End of program
```

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