



GLOBAL JOURNAL OF COMPUTER SCIENCE AND TECHNOLOGY

Volume 11 Issue 23 Version 1.0 December 2011

Type: Double Blind Peer Reviewed International Research Journal

Publisher: Global Journals Inc. (USA)

Online ISSN: 0975-4172 & Print ISSN: 0975-4350

The Computational Complexity of the Traveling Salesman Problem

By Craig Alan Feinstein

Abstract - In this note, we show that the Traveling Salesman Problem cannot be solved in polynomial-time on a classical computer.

GJCST Classification : F.2.2, F.2.3



Strictly as per the compliance and regulations of:



The Computational Complexity of the Traveling Salesman Problem

Craig Alan Feinstein

Abstract - In this note, we show that the Traveling Salesman Problem cannot be solved in polynomial-time on a classical computer.

Consider the following well-known NP-hard problem :

Traveling Salesman Problem - A traveling salesman starts at city 1, travels to cities 2, . . . , n-1 in any order that the salesman chooses, and then ends his trip in city n. Let us denote $\delta(i, j)$ to be the distance from city i to city j. The goal of the Traveling Salesman Problem is to find the minimum total distance possible for the traveling salesman to travel. There are no restrictions on the possible distances $\delta(i, j)$ between each of the cities other than the requirement that each $\delta(i, j)$ is a positive integer and $\delta(i, j) = \delta(j, i)$ [1, 3, 4].

We give a simple proof that no deterministic and exact algorithm can solve the Traveling Salesman Problem in $o(2^n)$ time:

For any nonempty subset $S \subseteq \{2, \dots, n\}$ and for any city $i \in S$, let us define $\Delta(S, i)$ to be the length of the shortest path that starts at city 1, visits all cities in the set $S - \{i\}$, and finally stops at city i. Then the Traveling Salesman Problem is equivalent to the problem of computing $\Delta(\{2, \dots, n\}, n)$. Clearly, $\Delta(\{i\}, i) = \delta(1, i)$ and

$$\Delta(S, i) = \min\{\Delta(S - \{i\}, j) + \delta(j, i) \mid j \in S - \{i\}\},$$

when $|S| \geq 2$.

This recursive formula cannot be simplified, so the fastest way to compute $\Delta(\{2, \dots, n\}, n)$ is to apply this recursive formula to $\Delta(\{2, \dots, n\}, n)$. Since this involves computing $\Delta(S, i)$ for all $\Theta(2^n)$ nonempty subsets $S \subseteq \{2, \dots, n\}$ and each $i \in S$, we obtain a lower bound of $\Theta(2^n)$ for the worst-case running-time of any deterministic and exact algorithm that solves the Traveling Salesman Problem.

This lower bound is confirmed by the fact that the fastest known deterministic and exact algorithm which solves the Traveling Salesman Problem was first published in 1962 and has a running-time of $\Theta^*(2^n)$ [2, 4].

REFERENCES REFERENCES REFERENCIAS

1. T.H. Cormen, C.E. Leiserson, and R.L. Rivest, Introduction to Algorithms, McGraw-Hill, 1990.
2. M. Held and R.M. Karp, "A Dynamic Programming Approach to Sequencing Problems", Journal of SIAM 10, pp. 196-210, 1962.
3. C.H. Papadimitriou and K. Steiglitz, Combinatorial Optimization: Algorithms and Complexity, Prentice-Hall, Englewood Cliffs, NJ, 1982.
4. G.J. Woeginger, "Open problems around exact algorithms", Discrete Applied Mathematics, 156(3): 397- 405, 2008.