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An Option Pricing Model That Combines Neural Network Approach and Black Scholes Formula

Dr. S. K.Mitra[1]

[1] Institute of Management Technology

Received: 12 April 2012 Accepted: 1 May 2012 Published: 15 May 2012

Abstract

The Black Scholes formula for theoretical pricing of options exhibits certain systematic biases, as observed prices in the market differs from the formula. A number of studies attempted to reduce these biases by incorporating a correction mechanism in the input data. Amongst non-parametric approaches used to improve accuracy of the model, Artificial Neural Networks are found as a promising alternative. The study made an attempt to improve accuracy of option price estimation using Artificial Neural Networks where all input parameters are adjusted by suitable multipliers. The values of these multipliers were determined using known data that minimises errors in valuation. The study was carried out using Nifty call option prices quoted on National Stock Exchange for the period 01-Jul 2008 to 30-Jun-11 covering three years.

Index terms— Artificial Neural Networks, Options pricing, Black Scholes formula

all option pricing formula developed by Black-Scholes (1973) was a landmark in the history of financial modelling and still remains a favoured model for theoretical valuation of options. However the comparison of observed prices for options and the theoretical valuations from the Black-Scholes formula has given rise to a large literature. One of the co-author of the model, **??**lack (1975) himself observed certain biases in the formula.

The reasons for difference in pricing are numerous. Though the model assumes a lognormal distribution of the stock prices, Benoit Mandelbrot (1963) observed that the asset price returns are highly leptokurtic (exhibit ?fat tails') as the actual returns from a stock show evidence of extreme movements more frequently than possible from a lognormal distribution. Few studies investigated on the nature of the underlying asset price process which differed from the lognormal Brownian motion. These models assumed that volatility of the stock price process is stochastic and investigation was directed to capture the varying nature of the volatility. Similarly, few studies have tried to explain biases of the model and attempted to adjust the parameters of the model to eliminate systematic biases correction but none seems to be complete **??**Robinstein, 1985). As no alternative closed form parametric solution better than the B-S model was found, a number of nonparametric approaches were tried out and Artificial Neural Network (ANN) based models are found as a promising alternative **??**Bennell and Sutcliffe, 2003).

Artificial Neural Networks offer several advantages ? Firstly, Artificial Neural Networks have the ability to recognize patterns from training data sets and display ability to discover relationships among inputs and outputs directly from the data. ? Secondly, Black-Scholes option pricing formulas use nonlinear functions. The ANN models are equipped to handle non-linearity by suitable version of a non-linear activation function. ? Thirdly, parametric models use complex functions to frame relationship and in many cases the out of sample performance is poor **??**Bakshi et.al., 1997). ? Fourthly, the markets are changing rapidly and unless a model has capability to constantly update its parameters based on changing market scenarios, the validity of the model in the long run is uncertain. ANN models have some capability to learn continuously from the data and revise the knowledge in its network weights. The present study is based on original work of Black and Scholes model on which the ANN concept is superimposed in such a way that each input parameter is modulated by a multiplier. These multipliers are allowed to change to build a better input-output relationship.

The paper organized follows. A brief literature survey on application of ANN in pricing stock options is presented in Section 2. A description of the Black and Scholes Option pricing formula and few issues related to measurement of volatility is given in section 3. An overview of neural network and development of an ANN model is presented in Section 4. The data and numerical analysis comparing performance of the ANN model with Black & Scholes model is produced in Section 5 and the paper is concluded in Section 6.

A large number of academic studies have examined the relative performance of ANNs in pricing equity options in several countries, few of the studies are mentioned here. Hutchinson et al. (1994) used three ANN models and compared their performance with the Black-Scholes model in American-style call options and found that the ANN models gave better results in comparison to Black-Scholes. Similarly, Geigle and Aronson (1999) studied the performance of ANN models in American-style options on S&P500 futures, and confirmed the superiority of Black-Scholes. Malliaris and Salchenberger (1993) also carried out a similar study but found that Black-Scholes model was better for inthe-money options, but the ANN models performed superior for out-of-the-money options. Ghaziri et al. (2000), Saito and Jun (2000) and many others compared the performance of ANN models in European-style call and concluded that the ANN models can give superior results compared to Black-Scholes. **??**ajbcygier et al. (1996) used three ANN models in pricing American-style call options on Australian Share Price Index futures and found that the ANN models were inferior to the theory-based Models in general but for near-the-money of short-maturity period, the ANN models were better. Many of these papers maintain the view that ANN models are capable of generating better results in comparison to closed-form models in pricing call options. In the present study the original Black and Scholes model is taken as the benchmark and the ANN concept of applying multipliers to the data is superimposed.

The original Black-Scholes (1973) formula uses five input parameters to price European style equity options. The Black-Scholes formulas for the prices of European Calls (C) and Puts (P) for no dividend paying stocks are **??**Hull, 2004)) (. .) (. 2 1 d N e X d N S C rt ? ? ?) (.) (. . 1 2 d N S d N e X P rt ? ? ? ? ? Where ? ? ? ? 2 1 ln / / 2 s x r dt t ? ? ?? ? 21 d d t ? ??

In this formula S = current price of the security X = Exercise price of option r = Risk free rate of interest t = Time to expiry of the option content ? = Volatility of the underlying asset N(x) is the cumulative probability function for a standardised normal variable. In other words, it is the probability that a variable with a standard normal distribution ? (0,1) will be less than x.

Among parameters described above, the standard deviation (?) of the returns during the life of the option can not be known in advance and consequently an estimate is required. There is no consensus on the appropriate method for estimating standard deviation of the price series. Further, it is a common knowledge that ??' of the price series varies with time. As a consequence very old data may not be appropriate for estimating the value of ?. According to **??**ull (2004), a compromise solution is to use closing daily prices of few recent months and converting the daily volatility to the annualised volatility as follows . annualised daily Trading days per annum

1 ?? ?

The number of trading days per year excluding weekly offs and holidays are usually taken 250 or 252. The historical volatility of a security is calculated as a standard deviation of a stock's returns over a fixed number of days. Choosing the appropriate period of observation is tricky. Longer period of observation by and large improve accuracy; however, it is found that volatility varies with time and very old data may not be relevant for predicting the future. Therefore, using a past period that is close to the validity period of the option is used by many investors.

As volatility is time varying, a time series approach can be used to measure and forecast ?. The J.P. Morgan RiskMetrics approach to estimating volatility uses an exponentially weighted moving average model (EWMA). The exponential moving average of historical observations allows capturing the dynamic features of volatility. The expected volatilities of a future period in the EWMA model are estimated using the following formula:? ? 2 2 2 11 1 n n n r ? ?? ? ?? ? ? ?

, where r n-1 is the return of the price series for the day (n-1). The return is obtained using natural logarithm of the price ratio from day n to day n-1, i.en n-1 n-1 p r = ln p ?? ?? ??

where p n is the actual price on day n. ? is a decay factor that determines the weight of past returns in comparison to immediate return. A further improvement of EWMA model is the application of ARCH-GARCH series of models An alternative method is to estimate a standard deviation that minimizes option pricing error in previous transaction and the measure is known as implied volatility. Many studies documented presence of systematic biases in implied volatility measures. **??**obinstein (1985) found that implied volatility is a The measure often exhibits a U-shape curve which is known as -volatility smile?.

2 Global

The absence of a unanimous procedure to estimate volatility to be used in the B-S model is a major hindrance as different measures give different option pricing. Nevertheless an estimate of ? is required as it is one of the important inputs for getting option value and practitioners use various approximations as per convenience.

The interest in neural networks emerged after the concept was introduced by McCulloch and Pitts **??**1943). Artificial neural networks (ANN) are used as generalizations of mathematical models of natural systems. The necessary processing elements of neural networks are termed as artificial neurons, or nodes. The basic structure of a neural network consists of three types of neuron layers: input, hidden, and output layers. In case of a feed-forward network, the flow of information is from input to output units, in a unidirectional manner. The data passes through the multiple nodes without any feedback of information. There are different types of neural network architectures, depending on the requirements of the application. In the study, a unidirectional feed-forward model was used. The flow of signal in the neural network model is illustrated in Figure 1, where the signal flow from inputs x 1 , . . . , x n and produce a final output ?O'.

Transmission of signals between neurons is facilitated using an activation functions which are useful for the input to determine the output from a neuron. This function tries to establish a relationship between the input variables and the output desired. The popular transfer functions are the sigmoid, the hyperbolic tangent, the Gaussian and their variants. The transfer function also helps to establish non linear relationships in the modeling.

In figure 1, the input signals are modified by multiplying a weight to each signal and the modified signals are added together to determine the combined strength of their output using the following activation function.1 n ii i O f x ? ? ?? ? ?? ?? ? Fig 2 : A Multilayer ANN

A multiplayer network consists of several layers are presented in figure **??**. The input variables are presented to the input layer of processing elements, which sends signal that propagates through the network layer by layer till it produce final output. The number of hidden neurons determines the complexities of information processing and influence how well the network is able to process the data. A large number of hidden neurons will ensure perfect input-output data matching by framing complex relationships. In such case the network will be capable of giving correct prediction from the trained dataset, but its performance on new data remains questionable. The network need to be trained in such a way that it retains the capacity to generalize the learning and can process new data as well. With too few hidden neurons, the network may not learn the relationships amongst training data but will fail to generalize its output in the out of sample data. Thus selection of the number of hidden neurons is an important decision. In this paper a simple feed-forward network, which is one of the common artificial network model, is used.

The knowledge of the network is supposed to be stored in the weights that are multiplied with the original signal strength. The weights are obtained by a process of adaptation using past data where both inputs and outputs are known. The adjustments of the weights are carried out using an iterative process. At each step in the process, small changes are introduced to the weights to bring the final outputs close to their desired values. This process is known as training the network and the set of examples as training set.

Initially, the weights are initialized to random values and for each set of training, the difference (error) between known output and network output is estimated. In the next step the weights of network are altered in such a manner that the sum of errors is minimised. The values of weights that minimises the squared error are the optimised weights of the trained network. After the completion of training the trained network can produce final output from a given input data set in the first layer.

The study aims to develop an ANN model that can improve difference between the theoretical option price and actual quoted price.

3 a) Network Inputs

The Black & Scholes model uses five parameters as inputs to estimate the theoretical option price. In the proposed model the same five parameters are used with a multiplying factor attached to each of the parameter. In absence of a standard procedure to measure volatility, ? was estimated by calculating standard deviations from past 60 day's returns. Although GARCH based methodologies can measure time varying volatilities in a better way, we expect the network to establish some kind of relationship from the training procedure and therefore relied on a simple estimation based on standard deviation. The volatility estimated on daily basis is annualized assuming 252 trading days in a year (Hull, 1999). The above input-output relationship for the ANN model closely resembles the original B-S model except that each parameter is multiplied by an adjusting weight. When weights (w 1 to w 7) are initialized to value of 1, the model gives output as expected from the original B-S model. While training the network, these weights (w 1 to w 7) will be altered so as to minimize difference between the network output and quoted option price.

The purpose of the study is to develop a model that improves accuracy of theoretical option pricing. In this study option prices are calculated using both Black and Scholes model and the proposed ANN model and results are compared.

4 a) Data

The valuation using Black-Scholes model requires values for six input parameters: spot price, strike price, maturity, risk-less interest rate, dividend rate and volatility. The closing values of the S&P CNX Nifty index series were collected from the website of National Stock Exchange of India www.nseindia.co.in for a three year period from 1 st July 2008 to 30 th June 2011.

The mis-pricing in the thinly traded options are supposed to be higher than in case of highly traded options and therefore only those options where daily volume exceeded more than 100 lots per day were short listed for the analysis.

To facilitate avoiding redundant observations, the last traded option each day was considered that is a particular combination of strike price and time to maturity. The sample contains 29724 option prices and has been divided into 12 groups of tree months each. The short listed database is further split into 12 quarterly groups as under.

5 From

To No. of Observation The accuracy of an option pricing model can be judged by comparing the actual market prices and theoretical valuation as per the chosen model.The differences between actual and theoretical values are errors of the model. A model that produces lowest error can be considered as a better model. There are several measures to compute errors, in the study, following estimates are used to measure errors.

Total Error (TE) is a sum of individual errors calculated as follows:1 N n n TE e ? ? ?

, where N is number of observation Mean Error (ME) is the arithmetic average of individual errors:1 1 N n n ME e N ? ? ?

Total squared error (TSE) computes the sum of the squared error values. This method of measuring error is commonly used statistical modelling. Compared to the total error value, this measure is very sensitive to large errors and penalises a model heavily that produce large error. Total Squared Error (TSE) can be computed as follows. compares the means of two error series. It computes the difference between the two error variables and tests whether the average difference of error is significantly different from zero. The null hypothesis is that there is no significant difference between the means of the two error series.

6 c) Error Analysis

At the beginning all adjusting weights (w 1 to w 7) were initialized with value of 1, and output from the model were identical to the option prices as per B-S model. The error analysis is given in table 2.

In the next step a training process was imparted to minimise the Total Squared Error by changing adjusting weights (w 1 to w 7). The total squared error in Black and Scholes model for the selected sample was 77,714,382. To carry out minimisation, an Add-in Solver program was used (details for using solver programme is available from ?help' menu in Microsoft Excel). The Solver parameters were set such that the cell address containing the formula for sum of squared errors were minimised, subject to changing contents of the cells that contain adjusting weights (w 1 to w 7). After several iterations, the solver gave a solution and the weights were optimised. The optimised weights of the ANN were given in table 3. These are the combination of weights that minimised total squared error.

When the optimal weights are found out (table 3) the weights are used for estimation of options and the error analysis is given in table 4. It may be observed from table 4 that the total squared error has substantially reduced from 77,714,382 to 19,622,082 (reduction of error by 74.75%). After the optimisation of weights the ANN model was capable to estimate option prices from new set of input data.

However, in table 4, the adjusting weights for each quarter were calculated using data for the same quarter. Use of these weights was again to estimate option prices using ANN model of the same quarter can give rise to in-sample bias. To eliminate this bias, weights calculated using input data for a particular quarter were used to generate option prices for the next quarter. For example, weights (w 1 to w 7) for the quarter of January-March were used to calculate option prices in the quarter of April-June and so on. Thus optimised weights obtained after training with input data for a period was used in estimation of option prices for next period. This procedure was repeated on a rolling basis for each period and errors were produced in table 5.

It may be seen from the table 5 that the total squared error for the period was 34,261,514,which was substantially lower than the Black and Scholes total squared error value of 77,714,382 (reduction of 55.91%)

The paired sample t-tests involving errors from Black and Scholes model and ANN model was produced in Table 6 and it was found that p-value was <0.01 and therefore difference of errors between the models were highly significant.

The classical biases found in the usual option pricing models motivated both researchers and practitioners to investigate alternative methods and ANN models were found to be a promising alternative. In the study a new model was conceived based on the original Black and Scholes model and the ANN concept of attaching multiplier weights to the data were introduced. It was found that model using the ANN approach has given superior results compared to original Black-Scholes model in pricing S&P CNX Nifty index call options.

The study initially estimated the differences between the actual call prices in the market, and theoretically estimated Black-Scholes call prices and used a training procedure that attempted minimizing the differences my altering values of the adjusting weights.

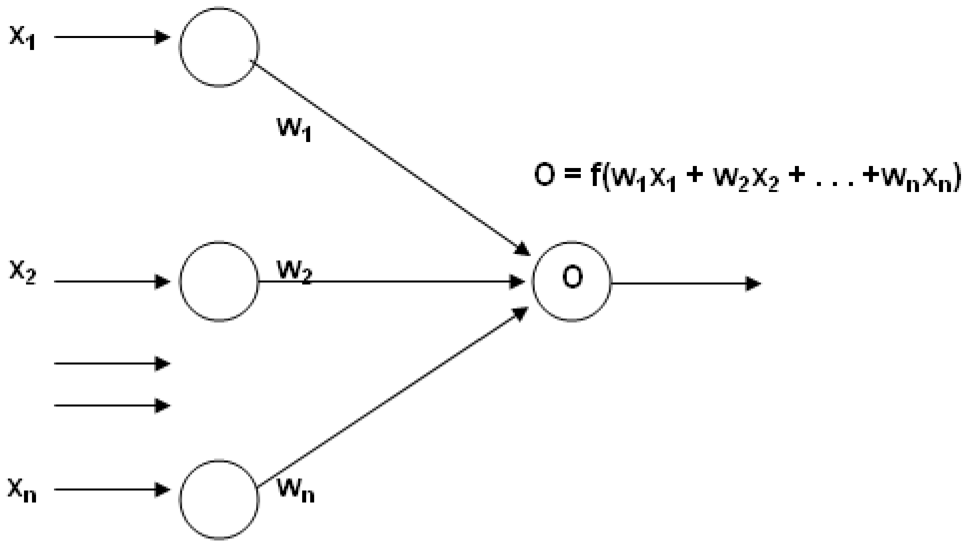
The differences in prices could also be the result of violations of some of the assumptions made in the derivation of the Black-Scholes model. For example, the original model assumed that the volatility and risk free interest rate were constant over the life of the option which is not true. The ANN model has a capacity to automatic adjustment of the changes in these variables by changing the adjusting weights.

Though the model is tested only on Call options of the index, it is expected that same can also be extended to other type of options. It is to be noted that the present study did not alter any assumption of the original model; it merely superimposed adjustment weights at each input and intermediate variable. These adjustment weights are allowed to vary so that the valuation errors are minimized. It was observed that use of the concept could reduce the total squared error by 55.91%.

Based on the observation, it may be commented that Artificial Neural Networks used in the study had some capability to develop relationship from exposure of past data and these relationships were stored in adjusting weights.

Further, markets were constantly changing and hence model needed constant updating. The ANN model used in the study was therefore trained and updated on quarterly intervals by altering the weights associated with it and produced output that were better [1 2]

Figure 1:



1

Figure 2: Fig 1 :

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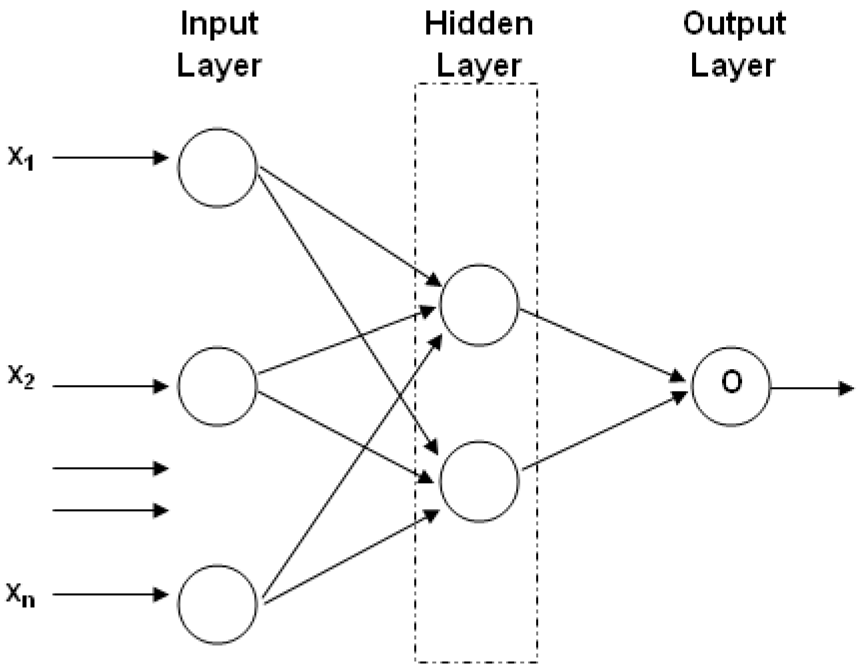

Figure 3:

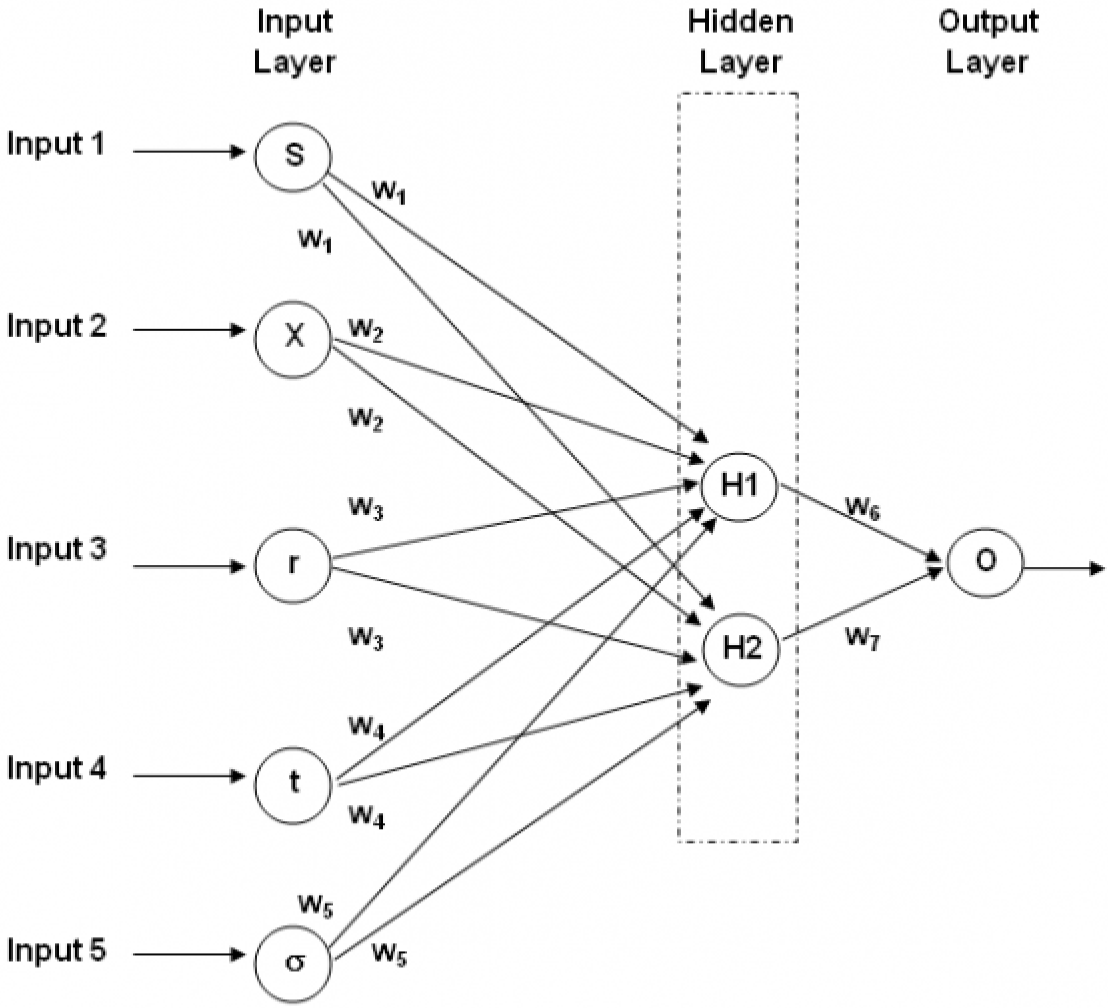

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Figure 4: Figure - 3 :

1

Sl	Parameter	Description
1	S	Spot price of the security
2	X	Exercise price of call option
3	r	Risk free rate of interest
4	t	Time left until option expiry (date in year fraction)
5	?	A measure of implied volatility (calculated as standard deviation of past 60 days daily return of underlying security)

Figure 5: Table 1 :

2

From	To	No. of Observation	Total Error	Mean Error	Total Squared Error	RMSE
1-Jul-08	30-Sep-08					
1-Oct-08	31-Dec-08	2337	-49495	-21	13097865	5605
1-Jan-09	31-Mar-09	1838	-25819	-14	3170830	1725
1-Apr-09	30-Jun-09	2173	-73642	-34	19644989	9040
1-Jul-09	30-Sep-09	1966	-16043	-8	2849732	1450
1-Oct-09	31-Dec-09	2021	7087	4	1531320	758
1-Jan-10	31-Mar-10	2249	-6882	-3	3001634	1335
1-Apr-10	30-Jun-10	2597	-39830	-15	5571740	2145
1-Jul-10	30-Sep-10	2954	9034	3	1715159	581
1-Oct-10	31-Dec-10	3088	6701	2	2217571	718
1-Jan-11	31-Mar-11	3417	-28735	-8	3054544	894
1-Apr-11	30-Jun-11	3116	-25311	-8	1449770	465
1-Oct-08	30-Jun-11	27756	-311699	-11	77714382	2615

Figure 6: Table 2 :

3

From	To	w1	w2	w3	w4	w5	w6	w7
1-Jul-08	30-Sep-08	1.0160	1.0407	0.8535	0.9366	0.7720	0.6808	0.6371
1-Oct-08	31-Dec-08	0.7434	0.7573	0.8565	0.8709	0.7446	0.7560	0.6969
1-Jan-09	31-Mar-09	1.0091	0.9925	0.7589	0.7165	0.9184	0.9281	0.9144
1-Apr-09	30-Jun-09	1.0051	0.9966	1.5000	1.1488	0.5172	0.9431	0.9246
1-Jul-09	30-Sep-09	1.0040	0.9921	0.8108	0.9209	0.9009	0.9468	0.9379
1-Oct-09	31-Dec-09	1.0003	0.9953	0.9997	0.9829	1.0200	0.9650	0.9604
1-Jan-10	31-Mar-10	1.0008	0.9928	0.7617	1.0110	0.9798	0.9182	0.9115
1-Apr-10	30-Jun-10	1.0145	1.0028	0.6606	0.9664	0.7785	0.9277	0.9208
1-Jul-10	30-Sep-10	1.0096	1.0145	1.1975	1.0393	1.0207	0.9591	0.9555
1-Oct-10	31-Dec-10	1.0033	0.9981	1.0065	0.9795	0.9460	0.9760	0.9710
1-Jan-11	31-Mar-11	1.0061	0.9991	0.9270	0.9438	0.9092	0.9813	0.9786
1-Apr-11	30-Jun-11	1.0017	0.9994	0.9531	0.9831	0.9546	0.9828	0.9832

From	To	No. of Observation	Total Error	Mean Error	Total Squared Error	RMSE
1-Jul-08	30-Sep-08					
1-Oct-08	31-Dec-08	2337	10218	4	2175413	931
1-Jan-09	31-Mar-09	1838	2715	1	722393	393
1-Apr-09	30-Jun-09	2173	14674	7	3097209	1425
1-Jul-09	30-Sep-09	1966	5590	3	1277338	650
1-Oct-09	31-Dec-09	2021	3227	2	1224967	606
1-Jan-10	31-Mar-10	2249	3860	2	1519347	676
1-Apr-10	30-Jun-10	2597	8681	3	1624959	626
1-Jul-10	30-Sep-10	2954	2003	1	1369816	464
1-Oct-10	31-Dec-10	3088	2117	1	1869777	605
1-Jan-11	31-Mar-11	3417	4649	1	2260257	661
1-Apr-11	30-Jun-11	3116	2932	1	986250	317
1-Oct-08	30-Jun-11	27756	68764	2	19622083	660

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Figure 7: Table 3 :

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Figure 8: Table 4 :

5

From	To	No. of Observation	Total Error	Mean Error	Total Squared Error	RMSE
1-Jul-08	30-Sep-08					
1-Oct-08	31-Dec-08	2337	44298	19	3687783	1578
1-Jan-09	31-Mar-09	1838	-13704	-7	2372613	1291
1-Apr-09	30-Jun-09	2173	-7361	-3	6313491	2905
1-Jul-09	30-Sep-09	1966	18629	9	4070658	2071
1-Oct-09	31-Dec-09	2021	13866	7	1622056	803
1-Jan-10	31-Mar-10	2249	-12846	-6	2219662	987
1-Apr-10	30-Jun-10	2597	-28040	-11	3526846	1358
1-Jul-10	30-Sep-10	2954	40681	14	2794633	946
1-Oct-10	31-Dec-10	3088	2628	1	2444599	792
1-Jan-11	31-Mar-11	3417	-23921	-7	2623744	768
1-Apr-11	30-Jun-11	3116	-918	0	1091071	350
1-Oct-08	30-Jun-11	27756	41410	1	34261514	1153

Figure 9: Table 5 :

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Paired Samples Test

	Paired Differences				
	Mean	Std. Error Std. Deviation	Mean	t	p-value (2-tailed)
B-S & ANN Model Errors	- 12.617	40.639	.236	- 53.527	.000

Figure 10: Table 6 :

[Mcculloch and Pitts ()] 'A logical calculus of the ideas immanent in nervous activity'. W S Mcculloch , W H Pitts . *Bulletin of Mathematical Biophysics* 1943. 5 p. .

[Malliaris and Salchenberger ()] 'A neural network model for estimating option prices'. M Malliaris , L Salchenberger . *Journal of Applied Intelligence* 1993. 3 (3) p. .

[Hutchison et al. ()] 'A non-parametric Approach to pricing and Hedging Derivative Securities via Learning Networks'. J M Hutchison , A W Lo , T Poggio . *Journal of Finance* 1994. 49 (3) p. .

[Geigle and Aronson ()] 'An artificial neural network approach to the valuation of options and forecasting of volatility'. D S Geigle , J E Aronson . *Journal of Computational Intelligence in finance* 1999. 7 (6) p. .

[Bennell and Sutcliffe] *Black-Scholes Versus Artificial Neural Networks in Pricing FTSE 100*, J Bennell , C Sutcliffe .

[Bakshi and Cao ()] 'Empirical Performance of Alternative Option Pricing Models'. G Bakshi , C Cao , Chen , Z . *Journal of Finance* 1997, 2003-2049. 52.

[Lajbcygier and Connor ()] 'Improved option pricing Using Artificial Neural Networks and Bootstrap Methods'. P Lajbcygier , J T Connor . *International Journal of Neural Systems* 1997. 8 (4) p. .

[Anders et al. ()] 'Improving the pricing of options: a neural network approach'. U Anders , O Korn , C Schmitt . *Journal of Forecasting* 1998. 17 (5-6) p. .

[Saito ()] 'Neural network option pricing in connection with the Black and Scholes model'. Jun S L Saito . *Proceedings of the Fifth Conference of the Asian Pacific Operations Research Society*, (the Fifth Conference of the Asian Pacific Operations Research SocietySingapore) 2000. 2000.

[Ghaziri et al. ()] 'Neural networks approach to pricing options'. H Ghaziri , S Elfakhani , J Assi . *Neural Network World* 2000. 10 (1-2) p. .

[Rubinstein (1976)] 'Non-parametric Tests of Alternate Option Pricing Models Using All Reported Trades and Quotes on the 30 Most Active CBOB option classes from'. M Rubinstein . *The Journal of Finances* 1995. August 23. 1976 through August 31. 1978. 1985. XL p. .

[Yao and Li ()] 'Option price forecasting using neural networks'. J Yao , Tan Y C L Li . *Omega* 2000. 28 (4) p. .

[Hull ()] *Options Futures and other Derivatives*, C J Hull . 1999. Prentice Hall. (4th Ed)

[Saxena] A Saxena . *Valuation of S&P CNX Nifty Options: Comparison of Black-Scholes and Hybrid ANN Model*,

[Black ()] 'The pricing of commodity contracts'. F Black . *Journal of Financial Economics* 1976. 3 p. .

[Black and Scholes ()] 'The Pricing of Options and Corporate Liabilities'. Black , M Scholes . *Journal of Political Economy* 1973. 8 p. .

[Mandelbrot ()] 'The Variation of Certain Speculative Prices'. B Mandelbrot . *The Journal of Business* 1963. 36 (4) p. .

[Working Paper downloaded from www Options ()] 'Working Paper downloaded from www'. *Options* 2003. (ssrn.com)