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Prolific Generation of Williamson Type Matrices

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Abstract

A new method of generating Williamson type Matrices A, B, C, D is described such that (i) A, B, C, D are symmetric. (ii) A, B, C are circulant matrices and D is a back circulant matrix. All such Williamsom type matrices of order n = 7, 9, 11, 13, 15, 17 are obtained by exhaustive computer search. The number of Williamson type Matrices constructed here is much greater than that of Williamson Matrices of same order. For example there are only 4 Williamson Matrices of order 17 but by our method we have obtained 504 Williamson type Matrices of order 17.

Index terms— Hadamard Matrices, , circulant and back circulant matrices, turnpike or partial digest problem.

1 INTRODUCTION

e recall the following definitions from Craigen and Kharaghani [1].

1.1 Hadamard Matrix [or H-Matrix] : An n x n (+1, -1) matrix H is a Hadamard matrix if HH T = nI n . It is conjectured that an H-matrix exists for every order n = 4t where t is a positive integer. 1 Back circulant bcirc (0 0 .... 0 1) is called back diagonal matrix.

1.5 Matrics used in the construction of H-Matrices : n x n (+1, -1) matrices A, B, C, D satisfying AA T + BB T + CC T + DD T = 4nI n (1) Are (i) Williamson Matrices if they are symmetric and circulant.

(ii) Goethals Seidel type matrices if they are circulant but not necessarily symmetric. (iii) Williamson type matrices if they are pairwise amicable. vide [1] 1.5 Matrics used in the construction of H-Matrices : n x n (+1, -1) matrices A, B, C, D satisfying AA T + BB T + CC T + DD T = 4nI n (**??**) are (i) Williamson Matrices if they are symmetric and circulant. (ii) Goethals Seidel type matrices if they are circulant but not necessarily symmetric. (iii) Williamson type matrices if they are pairwise amicable. vide [1] 1.6 Orthogonal Design OD (4t, t, t, t, t): OD (4t, t, t, t, t) is an orthogonal design of order 4t and type (t, t, t, t), t is a +ve integer, which is defined as an 4t x 4t matrix with entries ±A, ±B, ±C, ±D (A,B,C,D are commuting indeterminates) satisfying XX T = t(A 2 +B 2 +C 2 +D 2)I 4t For details vide Geramita and Seberry [2] II.

2 PREVIOUS WORK

If A,B,C,D are Williamson or Williamson type Matrices then the H-matrix, H can be constructed asA -B -C -D B A -D C H = C D A ?B D -C B A

Originally Williamson [3] constructed Williamson matrices for m ? 21, m=25, 37, 43. Baumert, Golomb and Hall [4] constructed Williamson matrix for m = 23. Baumert and Hall [5] found all solutions for 3 ? m ? 23 and some solutions for m =25, 27, 37, 43. For details of the solutions vide Hall [6]. Baumert [7] gave one solution for m = 29. Koukouvinos and Kounians [8] made exhaustive search for all Williamson atrices of order 33.

Williamson type martrices have been constructed by Seberry [9], [10] & Whiteman [11].

If A, B, C, D are circulant matrices satisfying equation (**??**) then H-matrix G can be obtained as the Goethals & Seidel array [12] A?BR ?CR ?DR BR A ?D T R C T R G = CR DR A -BR Where R is a (0,1) -back DR ?CR BR A diagonal matrix.

A Quadruple of Williamson type matrices A,B,C,D has advantage over other Quadruples used to construct H-matrices. The following lemma of Baumert and Hall [vide Colbourn & Dimtz [13] shows that from a quadruple(A,B,C,D) of Williamson type matrices. Several Hadamard matrices can be constructed.

Lemma 1: The existence of orthogonal design OD(4t; t,t,t,t) and four Williamson type matrices of order n implies the existence of H-matrices of order 4nt. Though it is generally conjectured that the above OD exists for all t, the existence is known for t ?73 (vide Colbourn and Dinitz ([14] p295).

3 III. METHODOLOGY

3.1 Some basic facts: We will begin with the following (new) definitions: (we assume that n is an odd positive integer) (i) Input Set A set S k ={n 1 , n 2 , ...n k } of inergers where 0 < n i < n, k is even, will be called an input set. The input set S k will be called symmetric if n i ?? S k ? n -n i ?? S k (ii) Output Vector Let m = (n-1)/2. Let S k be an input set defined above Let S k + j = { n 1+j , n 2+j , ...n k+j }, where + stands for addition mod n. Let r j = S k+j -S k = the order of the set (S k+j) -S k —-(**??**) and e j = n -4r j , j = 1, 2, 3 ....m —-(2) Binary reprentation of S k : row vector b k =(a 1 , a 2 , ...a n -1) will be called binary representation vector ((BR)-vector) of S k if a i = -1 if i ?? S k & a i = +1, otherwise

4 Method of Construction

Step-I Generation of size vector First construct 4-vector (k 1 , k 2 , k 3 ; k 4) which consists of feasible sizes of the four input sets as follows Express 4n as 4n = n Step-IV Sum of output vectors corresponding to three symmetric input sets Form the set S = {s=(s 1 , s 2 , s 3 , ... s m) : s is the sum of triplets of output vectors corresponding to symmetric input sets S k1 , S k2 , S k3 obtained in Step II (a) Omit all vectors ?? S for which |si| ? n -2. Let S' be the resulting set. Also record the correspondences (S k1 , S k2 , S k3) ? s ?? S' .

Step-V Set of output vectors corresponding to S k4

Form the set T of output vectors t = (t 1 , t 2 , t 3 , .. (b) In the multiset M of differences obtained in (a) i appears f i times i = 1, 2, 3, ... m, where f i are numbers defined in (ii) t ? T will be called feasible vector, if the set D 1 defined in (iii) exists. Each feasible t ? T will give Williamson type matrices A, B, C, D which can be obtained as follows Circulant A, B, C can be obtained through the correspondence : feasible t ?s ? {A, B, C} using the correspondence in Step-IV....... t m) of S k4 obtained in Step II (b) Let T'= {-t = (-t 1 , -t 2 , -t 3 , ... -t m)} Record all correspondences S k4 ? -t Step-VI

The back circulant matrix D can be obtained as follows : [15], [16], [17], [18]). Using the method described above, we have obtained all Williamson type matrices of order 9,11,13,15 & 17 by exhaustive computing search.(iv) if D 1 = (d 1 ,

IV.

5 RESULTS

Williamson type matrices of order 9

Type -I (4 x 9 = 1 2 + 1 2 + 3 2 + 5 2) Subtype -I (Size Vector (4, 2, 6; 4)) Sl.no Input Set Set of Williamson type Matrices Output Vector 1 {3,6} A circ (1 1 1 -1 1 1 -1 1 1) 1 1 5 1 {1,4,5,8} B circ (1 -1 1 1 -1 -1 1 1 -1) -3 -3 1 1 {2,3,4,5,6,7} C circ (1 1 -1 -1 -1 -1 -1 -1 1) 5 1 -3 -3 {3,5,7,8} D circ (1 1 1 -1 1 -1 1 -1 -1) -3 1 -3 1 2 {3,6} A circ (1 1 1 -1 1 1 -1 1 1) 1 1 5 1 {1,2,7,8} B circ (1 -1 -1 1 1 1 1 -1 -1) 1 -3 1 -3 {1,3,4,5,6,8} C circ (1 -1 1 -1 -1 -1 -1 1 -1) -3 5 -3 1 {3,6,7,8} D circ (1 1 1 -1 1 1 -1 -1 -1) 1 -3 -3 1 3 {3,6} A circ (1 1 1 -1 1 1 -1 1 1) 1 1 5 1 {2,4,5,7} B circ (1 1 -1 1 -1 -1 1 -1 1 1) -3 1 1 -3 {1,2,3,6,7,8} C circ (1 -1 -1 -1 1 1 -1 -1 -1) 1 -3 -3 5 {4,6,7,8} D circ (1 1 1 1 -1 1 -1 -1 -1) 1 1 -3 -3 Subtype-II (Size Vector (2, 4, 4; 3)) Sl.no Input Set Set of Williamson type Matrices Output Vector 1 {2,7} A circ (1 1 -1 1 1 1 1 -1 1) 1 1 1 5 {1,4,5,8} B circ (1 -1 1 1 -1 -1 1 1 -1) -3 -3 1 1 {3,4,5,6} C circ (1 1 1 -1 -1 -1 -1 1 1) 5 1 -3 -7 {3,6,8} D circ (1 1 1 -1 1 1 -1 1 -1) -3 1 1 1 2 {4,5} A circ (1 1 1 1 -1 -1 1 1 1) 5 1 1 1 {1,3,6,8} B circ (1 -1 1 -1 1 1 -1 1 -1) -7 5 -3 1 {1,2,7,8} C circ (1 -1 -1 1 1 1 1 -1 -1) 1 -3 1 -3 {4,7,8} D circ (1 1 1 1 -1 1 1 -1 -1) 1 -3 1 1 3 {1,8} A circ (1 -1 1 1 1 1 1 1 -1) 1 5 1 1 {2,4,5,7} B circ (1 1 -1 1 -1 -1 1 -1 1) -3 1 1 -3 {2,3,6,7} C circ (1 1 -1 -1 1 1 -1 -1 1) 1 -7 -3 5A circ (1 1 1 -1 1 -1 -1 1 -1 1 1) -1 3 3 -5 -1 {1,2,3,8,9,10} B circ (1 -1 -1 -1 1 1 1 1 -1 -1 -1) 3 -1 -5 -1 -1 {1,2,3,5,6,8,9,10} C circ (1 -1 -1 -1 1 -1 -1 1 -1 -1 -1) -1 -1 3 7 -1 {4,6,9,10} D circ (1 1 1 1 -1 1 -1 1 1 1 -1 -1) -1 -1 -1 -1 3 2 {2,4,7,9} A circ (1 1 -1 1 -1 1 1 -1 1 -1 1) -5 3 -1 -1 3 {2,3,5,6,8,9} B circ (1 1 -1 -1 1 -1 -1 1 -1 -1 1) -1 -5 3 -1 -1 {2,3,4,5,6,7,8,9} C circ (1 1 -1 -1 -1 -1 -1 -1 -1 -1 1) 7 3 -1 -1 -1 {3,7,9,10} D circ (1 1 1 -1 1 1 1 -1 1 -1 -1) -1 -1 -1 3 -1 3 {3,4,7,8} A circ (1 1 1 -1 -1 1 1 1 -1 -1 1 1) 3 -5 -1 3 -1 {1,4,5,6,7,10} B circ (1 -1 1 1 -1 -1 -1 -1 1 1 1 -1) -1 -1 -1 -5 -3 {1,3,4,5,6,7,8,10} C circ (1 -1 1 -1 -1 -1 -1 -1 -1 1 1 -1) -1 7 -1 3 -1 {4,7,8,10} D circ (1 1 1 1 -1 1 1 1 -1 -1 1 1 -1) -1 -1 3 -1 -1 4 {1,2,9,10} A circ (1 -1 -1 1 1 1 1 1 1 -1 -1) 3 -1 3 -1 -5 {1,3,4,7,8,10} B circ (1 -1 1 -1 -1 1 1 1 -1 -1 1 1 -1) -5 -1 -1 3 -1 {1,2,3,4,7,8,9,10} C circ (1 -1 -1 -1 -1 1 1 1 -1 -1 -1 -1) 3 -1 -1 -1 7 {5,7,9,10} D circ (1 1 1 1 1 -1 1 -1 1 -1 -1) -1 3 -1 -1 -1 5 {1,5,6,10} A circ (1 -1 1 1 1 -1 -1 1 1 1 1 -1) -1 -1 -5 3 3 {2,4,5,6,7,9} B circ (1 1 -1 1 -1 -1 -1 -1 1 -1 1) -1 3 -1 -1 -5 {1,2,4,5,6,7,9,10} C circ (1 -1 -1 1 -1 -1 -1 -1 1 -1 -1) -1 -1 7 -1 3 {5,8,9,10} D circ (1 1 1 1 1 -1 1 1 -1 -1 -1) 3 -1 -1 -1 -1A circ (1 -1 1 1 1 -1 -1 1 1 1 1 -1) -1 -1 -5 3 3 {4,5,6,7} B circ (1 1 1 1 -1 -1 -1 -1 1 1 1) 7 3 -1 -5 -5 {1,2,3,5,6,8,9,10} C circ (1 -1 -1 -1 1 -1 -1 1 -1 -1 -1) -1 -1 3 7 -1 {2,4,7,9,10} D circ (1 1 -1 1 -1 1 1 -1 1 -1 -1) -5 -1 3 -5 3 2 {2,4,7,9} A circ (1 1 -1 1 -1 1 1 -1 1 -1 1) -5 3 -1 -1 3 {3,4,7,8} B circ (1 1 1 -1 -1 1 1 -1 -1 1 1) 3 -5 -1 3 -1 {2,3,4,5,6,7,8,9} C circ (1 1 -1 -1 -1 -1 -1 -1 -1 -1 1) 7 3 -1 -1 -1 {3,6,8,9,10} D circ (1 1 1 -1 1 1 -1 1 -1 -1 -1) -5 -1 3 -1 -1 3

{2,3,8,9} A circ (1 1 -1 -1 1 1 1 1 -1 -1 1) 3 -5 -5 -1 7 {1,2,9,10} B circ (1 -1 -1 1 1 1 1 1 1 -1 -1) 3 -1 3 -1 -5 {1,3,4,5,6,7,8,10} C circ (1 -1 1 -1 -1 -1 -1 -1 -1 1 -1) -1 7 -1 3 -1 {3,6,8,9,10} D circ (1 1 1 -1 1 1 -1 1 -1 -1 -1) -5 -1 3 -1 -1 4 {1,4,7,10} A circ (1 -1 1 1 -1 1 1 -1 1 1 -1) -5 -1 7 -5 3 {3,4,7,8} B circ (1 1 1 -1 -1 1 1 -1 -1 1 1) 3 -5 -1 3 -1 {2,3,4,5,6,7,8,9} C circ (1 1 -1 -1 -1 -1 -1 -1 -1 -1 1) 7 3 -1 -1 -1 {3,5,7,9,10} D circ (1 1 1 -1 1 -1 1 -1 1 -1 -1) -5 3 -5 3 -1 5 {2,4,7,9} A circ (1 1 -1 1 -1 1 1 -1 1 -1 1) -5 3 -1 -1 3 {4,5,6,7} B circ (1 1 1 1 -1 -1 -1 -1 1 1 1) 7 3 -1 -5 -5 {1,2,3,5,6,8,9,10} C circ (1 -1 -1 -1 1 -1 -1 1 -1 -1 -1) -1 -1 3 7 -1 {3,4,7,9,10} D circ (1 1 1 -1 -1 1 1 -1 1 -1 -1) -1 -5 -1 -1 3A circ (1 1 1 -1 1 -1 -1 1 -1 1 1) -1 3 3 -5 -1 {3,4,7,8} B circ (1 1 1 -1 -1 1 1 -1 -1 1 1) 3 5 -1 3 -1 {1,3,4,5,6,7,8,10} C circ (1 -1 1 -1 -1 -1 -1 -1 -1 1 -1) -1 7 -1 3 -1 {3,4,7,9,10} D circ (1 1 1 -1 -1 1 1 -1 1 -1 -1) -1 -5 -1 -1 3 7 {3,5,6,8} A circ (1 1 1 -1 1 -1 -1 1 -1 1 1) -1 3 3 -5 -1 {2,3,8,9} B circ (1 1 -1 -1 1 1 1 1 -1 -1 1) 3 -5 -5 -1 7 {1,3,4,5,6,7,8,10} C circ (1 -1 1 -1 -1 -1 -1 -1 -1 1 -1) -1 7 -1 3 -1 {3,6,7,9,10} D circ (1 1 1 -1 1 1 1 -1 -1 1 -1 -1) -1 -5 3 3 -5 8 {1,4,7,10} A circ (1 -1 1 1 -1 1 1 -1 1 1 -1) -5 -1 7 -5 3 {1,5,6,10} B circ (1 -1 1 1 1 -1 -1 1 1 1 -1) -1 -1 -5 3 3 {2,3,4,5,6,7,8,9} C circ (1 1 -1 -1 -1 -1 -1 -1 -1 -1 1) 7 3 -1 -1 -1 {3,6,7,8,10} D circ (1 1 1 -1 1 1 -1 -1 -1 1 -1) -1 -1 -1 3 -5 9 {2,4,7,9} A circ (1 1 -1 1 -1 1 1 -1 1 -1 1) -5 3 -1 -1 3 {1,2,9,10} B circ (1 -1 -1 1 1 1 1 1 1 -1 -1) 3 -1 3 -1 -5 {1,2,3,4,7,8,9,10} C circ (1 -1 -1 -1 -1 1 1 -1 -1 -1 -1) 3 -1 -1 -1 7 {3,6,7,8,10} D circ (1 1 1 -1 1 1 -1 -1 -1 1 1 -1) -1 -1 -1 3 -5 10 {2,5,6,9} A circ (1 1 -1 1 1 -1 -1 1 1 -1 1) -1 -5 3 7 -5 {3,5,6,8} B circ (1 1 1 -1 1 -1 -1 1 -1 1 1) -1 3 3 -5 -1 {1,2,3,4,7,8,9,10} C circ (1 -1 -1 -1 -1 1 1 -1 -1 -1 -1) 3 -1 -1 -1 7 {4,6,8,9,10} D circ (1 1 1 1 -1 1 -1 1 -1 -1 -1) -1 3 -5 -1 -1 11 {1,5,6,10} A circ (1 -1 1 1 1 -1 -1 1 1 1 -1) -1 -1 -5 3 3 {1,2,9,10} B circ (1 -1 -1 1 1 1 1 1 1 -1 -1) 3 -1 3 -1 -5 {1,2,4,5,6,7,9,10} C circ (1 -1 -1 1 -1 -1 -1 -1 1 -1 -1) -1 -1 7 -1 3 {4,6,8,9,10} D circ (1 1 1 1 -1 1 -1 1 -1 -1 -1) -1 3 -5 -1 -1 12 {1,3,8,10} A circ (1 -1 1 -1 1 1 1 1 -1 1 -1) -5 7 -5 3 -1 {1,2,9,10} B circ (1 -1 -1 1 1 1 1 1 1 1 -1 -1) 3 -1 3 -1 -5 {1,2,**4,5,6,7,9,10} C circ (1 -1**-1 1 -1 -1 -1 1 1 -1 -1) -1 -1 7 -1 3 {4,5,8,9,10} D circ (1 1 1 1 -1 -1 1 1 -1 -1 -1) 3 -5 -5 -1 3 13 {1,3,8,10} A circ (1 -1 1 -1 1 1 1 1 -1 1 -1) -5 7 -5 3 -1 {3,4,7,8} B circ (1 1 1 -1 -1 1 1 -1 -1 1 1) 3 -5 -1 3 -1 {1,2,4,5,6,7,9,10} C circ (1 -1 -1 1 1 -1 -1 -1 -1 1 -1 -1) -1 -1 7 -1 3 {4,7,8,9,10} D circ (1 1 1 1 -1 1 1 -1 -1 -1 -1) 3 -1 -1 -5 -1 14 {1,5,6,10} A circ (1 -1 1 1 1 -1 -1 1 1 1 -1) -1 -1 -5 3 3 {3,5,6,8} B circ (1 1 1 -1 1 -1 -1 1 -1 1 1) -1 3 3 -5 -1 {1,2,3,5,6,8,9,10} C circ (1 -1 -1 -1 1 -1 -1 1 -1 -1 -1) -1 -1 3 7 -1 {4,7,8,9,10} D circ (1 1 1 1 -1 1 1 -1 -1 -1 -1) 3 -1 -1 -5 -1 15 {2,4,7,9} A circ (1 1 -1 1 -1 1 1 -1 1 -1 1) -5 3 -1 -1 3 {2,5,6,9} B circ (1 1 -1 1 1 -1 -1 1 1 -1 1) -1 -5 3 7 -5 {1,2,3,4,7,8,9,10} C circ (1 -1 -1 -1 -1 1 1 -1 -1 -1 -1) 3 -1 -1 -1 7 {5,7,8,9,10} D circ (1 1 1 1 1 -1 1 -1 -1 -1 -1) 3 3 -1 -5 -5

6 Table IV WILLIAMSON TYPE MATRICES OF ORDER 13

Type -I (4 x 13 = 1 2 + 1 2 + 1 2 + 7 A circ (1 -1 -1 1 1 1 -1 -1 1 1 1 -1 -1) 1 -7 -3 1 5 -3 {3,5,6,7,8,10} B circ (1 1 1 -1 1 -1 -1 -1 -1 1 -1 1 1) 1 5 1 -3 -3 -7 {1,2,3,4,5,8,9,10,11,12} C circ (1 -1 -1 -1 -1 -1 1 1 1 -1 -1 -1 -1) 5 1 1 1 1 9 {2,5,7,9,11,12} D circ (1 1 -1 1 1 -1 1 -1 1 -1 1 -1 -1) -7 1 1 1 -3 1 Sl.no Input Set Set of Williamson type Matrices Output Vector 1 {1,5,8,12} A circ (1 -1 1 1 1 -1 1 1 -1 1 1 1 -1) -3 1 1 5 -3 5 {5,6,7,8} B circ (1 1 1 1 1 -1 -1 -1 -1 1 1 1 1) 9 5 1 -3 -3 -3 {1,2,6,7,11,12} C circ (1 -1 -1 1 1 1 -1 -1 1 1 1 -1 -1) 1 -7 -3 1 5 -3 {2,4,6,9,11,12} D circ (1 1 -1 1 -1 1 -1 1 1 -1 1 -1 -1) -7 1 1 -3 1 1

Type -III (4 x 13 = 3 2 + 3 2 + 3 2 + 5 A circ(1 -1 1 -1 -1 1 -1 -1 1 -1 -1 1 -1) -7 1 5 -3 1 1 {2,3,4,6,7,9,10,11} B circ(1 1 -1 -1 -1 1 -1 -1 1 -1 -1 -1 1) 1 -3 -3 1 1 1 {2,3,4,6,7,9,10,11} C circ(1 1 -1 -1 -1 1 -1 -1 1 -1 -1 -1 1) 1 -3 -3 1 1 1 (8,10,11,12} D circ(1 1 1 1 1 1 1 1 -1 1 -1 -1 -1) 5 5 1 1 -3 -3A circ (1 -1 -1 1 1 -1 1 1 1 1 1 -1 1 1 -1 -1) -1 -5 7 3 -5 -1 -1 {2,3,4,6,7,8,9,11,12,13} B circ (1 1 -1 -1 -1 1 -1 -1 -1 -1 1 -1 -1 -1 1) 1 -7 -3 1 5 -3 {1,2,3,4,5,10,11,12,13,14} C circ (1 -1 -1 -1 -1 -1 1 1 1 1 -1 -1 -1 -1 -1) 7 3 -1 -5 -5 3 3 {2,4,6,9,11,13,14} D circ (1 1 -1 1 -1 1 -1 1 1 -1 1 -1 1 -1 -1) -9 3 -1 -1 3 -5 3

Type -II (4 x 15 = 1 2 + 1 2 + 3 2 + 7 4,6,7,8,9,11,12 C circ (1A circ (1 1 -1 1 -1 1 1 -1 -1 1 1 -1 1 -1 1) -5 -1 -1 3 -1 7 -5 {1,2,6,7,8,9,13,14} B circ (1 -1 -1 1 1 1 -1 -1 -1 -1 1 1 1 -1 -1) 3 -5 -5 -5 -1 -1 7 {3,1 1 -1 -1 1 -1 -1 -1 -1 1 -1 -1 1 1) 3 -1 3 -1 -1 -5 -5 {7,10,12,14} D circ (1 1 1 1 1 1 -1 1 1 -1 1 -1 1 -1) -1 7 3 3 3 -1 3

7 WILLIAMSON TYPE MATRICES OF ORDER 17

Type -I (4 x 17 = 1 2 + 3 2 + 3 2 + 7 V.A circ(1 -1 1 1 1 -1 1 -1 -1 -1 -1 1 -1 1 1 1 -1) -3 5 -3 1 -7 1 -3 1 {1,2,3,6,8,9,11,14,15,16} B circ (1 -1 -1 -1 1 1 -1 1 -1 -1 1 -1 1 1 -1 -1 -1) -3 -3 1 -3 5 -3 1 1 {1,2 ,4,5,6,11,12,13,15,6} C circ(1 -1 -1 1 -1 -1 -1 1 1 1 1 -1 -1 -1 1 -1 -1) 1 -3 1 -3 -3 1 5 -3 {2,6,7,8,11} D circ(1 1 -1 1 1 1 -1 -1 -1 1 1 1 -1 1 1 1 1 1)**5**A circ(1 1 -1 1 1 1 -1 1 -1 -1 1 -1 1 1 1 -1 1) -3 1 1 5 -3 1 1 1 {1,3,4,7,8,9,10,13,14,16} B circ (1 -1 1 -1 -1 1 1 -1 -1 -1 -1 1 1 -1 -1 1 -1) -3 -3 -3 1 1 5 1 -3 {1,2,3,6,8,9,11,14,15,16} C circ(1 -1 -1 -1 1 1 -1 1 -1 -1 1 -1 1 1 -1 -1 -1) -3 -3 1 -3 5 -3 1 1 {2,3,4,5,7,8,9,10,11,12,13} D circ(1 1 -1 -1 -1 -1 1 -1 -1 -1 -1 -1 -1 -1 1 1 1) 9 5 1 -3 -3 -3 -A circ (1 -1 1 1 -1 1 1 1 -1 -1 1 1 1 -1 1 1 -1) -3 -3 1 1 9 -7 1 5 {1,3,8,9,14,16} B circ(1 -1 1 -1 1 1 1 1 -1 -1 1 1 1 1 -1 1 -1) -3 5 -7 1 1 5 1 1 {1,2,3,4,5,12,13,14,15,16} C circ(1 -1 -1 -1 -1 -1 1 1 1 1 1 1 -1 -1 -1 -1 -1) 9 5 1 -3 -7 -3 -3 -3 {2,6,7,10,12,13,16} D circ(1 1 -1 1 1 1 -1 -1 1 1 -1 1 -1 -1 1 1 -1) -3 -7 5 1 -3**5**A circ(1 1 1 -1 1 -1 1 1 -1 -1 1 1 -1 1 -1 1 1) -3 1 1 1 5 -3 1 {1,2,3,6,8,9,11,14,15,16} B circ(1 -1 -1 -1 1 1 -1 1 -1 -1 1 -1 1 1 -1 -1 -1) -3 -3 1 -3 5 -3 1 1 {1,5,6,7,8,9,10,11,12,16} C circ(1 -1 1 1 1 -1 -1 -1 -1 -1 -1 -1 -1 1 1 1 -1)**5**

8 REMARK

Remark-1 In the above tables A, B, C are circulant matrices and D is a back circulant matrix whose 1 st row is shown.

Remark-2 Table **??** shows that there exists a Williamson type matrix corresponding to the expression 4x19 = 1 2 + 5 2 + 5 2 + 5 2 , whereas there is no Williamson matrix to the above expression. This indicates that one can find Williamson type matrices by our method, where Williamson's method fails.

The following Table **??** shows that the number of Williamson type matrices of small order obtained by our method is much greater than that of Williamson matices of the same order.

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Figure 1:

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a 1 a 2 .. a n?1 a n
a n a 1 a 2 ... a n?1
called circulant matrix.
a n?1 a n a 1 ... a n?2
.. ..
a 2 a 3 .. .. a 1
1.4 Back circulant matrix : bcirc(a 1 , a 2 , ...a n)
is the matrix

a 1	a 2	.. a n?1		a n
a 2	a 3	..	a n	a 1
a 3	a 4	..	a 1	a 2
..	..	..	..	..
a n	a 1	..	..	a n-1

called back circulant matrix.

Figure 2:

then form a back circulant matrix D where first row contains -1 at d 1 th , d 2 th , ... d k th place and +1 elsewhere.

Remark : Step-(iii) is equivalent to turnpike or partial digest problem (vide

Figure 3:

III

Subtype-II (Size Vector (4, 4, 8; 5))

Sl.no	Input Set	Set of Williamson type Matrices	Output Vector
1	{1,5,6,10}		

Figure 4: Table III

V

Type -I (4 x 15 = 1 2 + 3 2 + 5 2 + 5 2)
Subtype-I (Size Vector (6, 10, 10; 7))

Sl.no	Input Set	Set of Williamson type Matrices	Output Vector
1	{1,2,5,10,13,14}		

Figure 5: Table V

VII

Type -I (4 x 19 = 1 2 + 5 2 + 5 2 + 5 2)
Subtype-I (Size Vector (8, 8, 8; 6))

Sl. no	Input Set	Set of Williamson type Matrices	Output Vector

Figure 6: Table VII

.1 May

??———e

where k 4 = k4 if k4 is odd Proof (i) In AA T e j = scalar product of 1 st row R1 of A with (j + 1) th row R j+1 of A . Let S k1 be the input set corresponding to 1 st row R1of A and j+S k1 be that corresponding to (j+1) throw R j+1 of A (where + stands for addition mod n). Let the order of the set (j + S k1) -S k1 be r j . The rows R 1 and R j+1 of A differ at 2r j places. Hence the scalar product R 1 R 2 is (n -2r 2) -2r 2 = n -4rj= e j by definition. By the same argument we get expressions for BB T , CC T & D 1 D 1 T . Since from (4) the sum of output vectors for A, B, C, D 1 is 0, it follows that ?AA T = circ (4n, 0, 0, ———-0) A, B, C, D 1 = 4nI n Proof (ii) Consider all k 1 (k 1 -1) differences (mod n) of the set S k1 . Suppose in the multiset of differences j appears g j times j = 1, **??** , ——-(n-1).

) Also j + S k1 and S k1 has g j common elements. Hence |(j + S k1) ? S k1 |= k 1g j =r j Also e j = n -4r j [from (**??**

.2 FUTURE WORK

Like Williamson's method, the present method requires great computational effort. However using genetic algorithm or some other heuristic method one can find some Williamson type matrices of higer order.

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