Growth Option Model For Oil Field Valuation

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Abstract

This paper considers the use of Real Option Approach (ROA) to value an oil field project. The Geometric Brownian Motion and the classic Black-Schole?s model is used to obtain the value of the fair price (option value F). We show that ROA is an invaluable tool in decision making in situations where investment involves high risk and uncertainty.

***Index terms*—** Real options, Brownian motion.

1 INTRODUCTION

??yers (1984)

first used the term "real options" to describe corporate investment opportunities that resemble options. He proposed that the value of a firm could be divided into the value of its assets in place and the value of these "future growth options". Growth options are also frequently referred to as expansion options.

Real options analysis (ROA) can more accurately model the nature of the investment and therefore provide a better basis for the investment decision. For example, ROA is preferred when the investment decision hinges on the outcome of a future event, or when there is enough uncertainty to defer the investment decision. Also companies use real options analysis when future growth is a significant source of the investment's value, when a traditional discounted cash flow (DCF) analysis returns a low or slightly negative net present value, and when the options associated with the investment could change management's decision from "no go" to "go". Essentially, real options analysis is ideal for companies in high-growth industries where there is a great deal of uncertainty and the investments are both large and strategic. High tech, venture capital, pharmaceutical and oil exploration all qualify, and interestingly all are early adopters of real options analysis.

The need to developing valuation models that is capable of capturing such features of investment as irreversibility, uncertainty as well as timing flexibility has resulted in a vast amount of literature on real options and investment under uncertainty. In his seminar paper Myers (1977) draws attention to the optimal exercise strategies of real options as being the significant source of corporate value. Brennan and Schwartz (1985) are one of the first to adopt the modern option pricing techniques (see Scholes, 1973 andMerton, 1973) to evaluate natural resource investments. The price of the commodity is used as an underlying stochastic variable About-University of Science and Technology Ifaki, Nigeria E-mail-topsmatic@yahoo.com upon which the value of the investment project is contingent. McDonald and Siegel (1986) derive the optimal exercise rule for a perpetual investment option when both the value of the project and the investment costs follow correlated geometric Brownian motions. The authors show that for realistic values of model parameters, it can be optimal to wait with investing until the present value of the project exceeds the present cost of investment by a factor of 2. This reflects substantial value of waiting in the presence of irreversibility and uncertainty. Majd and Pindyck (1987) contribute to the literature by considering the effect of a time to build on the optimal exercise rule. The optimal choice of the project's capacity is analyzed by Pindyck (1988) and Dangl (1999). **??**ixit (1989) analyzes the effects of uncertainty on the magnitude of hysteresis in the models with entry and exist. Dixit and Pindyck (1996) present a detailed overview of this early literature and constitute an excellent introduction to the techniques of dynamic programming and contingent claims analysis, which are widely applicable in the area of real options and investment under uncertainty. An introduction to real options, which is closer in the spirit to the financial options theory, is presented by **??**rigeorgis (1996).

There is the need for a good and reliable option-pricing model that will yield or give the best result. Therefore, the first reliable option-pricing model was derived by Black and Scholes (1973). The Black-Scholes formula can be used to obtain the value of European call options on non-dividend paying assets. The value of the European put with identical parameters can be inferred from the call value. Merton (1973) developed an option pricing formula for dividend-paying assets and made other significant contributions to the development of option pricing theory. Merton and Scholes won the Noble Price in Economics for their contributions to derivative pricing in 1997. Cox, Ross and Rubinstein (1979) built on the insights of Black and Scholes (1973) and others to develop the binomial option-pricing model. The binomial model is simpler to understand and explain than the Black-Scholes model, it is more widely used in practice, and is capable of generating the same results as the Black-Scholes. **??**rnold and Crack (2000) extended the binomial model to yield additional probabilistic information about the option that cannot be obtained directly from the Cox, Ross and Rubinstein (1979) model. It must be stressed that the interest here is more on the Black-Scholes model and so, we shall be examining and employing the Black-Scholes model.

2 BROWNIAN MOTION

For a project value V or the value of the developed reserve that follows a Geometric Brownian Motion, the stochastic equation for its variation with the time t is???? = ???????? + ????????**(1)**

where ???? = ???????????? ??????????????????? = ???????. ?? is the normal standard distribution, ?? is the drift and ?? is the volatility of V. In real options problems, there is a dividend like income stream ?? for the holder of the asset. This dividend yield is related to the cash flows generated by the assets in place. For commodities prices, this is called convenience yield or rate of return of shortfall. In all cases, the equilibrium requires that the total expected return ?? to be the sum of expected capital gain plus the expected dividend, so that ?? = ?? + ?? so that equation (1 Letting ?? = ???? ?? and using Ito's Lemma, we find that v follows the arithmetic (ordinary) Brownian motion:???? = ??(???? ??) = ??? ? ?? ?1 2 ?? 2 ? ?? + ?????? So ???? = ?? ? ???? + ????

Although, the volatility term is the same of the geometric Brownian for V, ??(???? ??) is different from ???? ?? ? due to drift. In reality, by the Jensen's inequality, ??(???? ??) < ???? ?? ? (Ito's effect).

III.

3 Real Options in Petroleum

A simple real option method is to exploit the power of the analogy with financial European call option on a stock paying a continuously compound dividend yield. In the analogy with petroleum, instead of the stock, the underlying asset is the developed reserve value, V (which is a function of petroleum prices). The excise price is the cost of development, D and the time to expiration, T is the relinquishment requirement.

Study has shown that there is high correlation between oil price, P and the market value of the developed reserve V, so it is reasonable to set V as a proportion of P. Let ?????? = ð ??”ð ??”???????? + ?????? - ð ??”ð ??”????????**(4)**

For a model which the rate of return on the developed reserve follows a geometric Brownian motion.?????? ???? = ?????? + ?????? (5) ð ??”ð ??”???????? + ?????? ? ð ??”ð ??”???????? ???? = ?????? + ?????? ? ???? = (?? ? ??)?? ???? + ????????**(6)**

This is a very important result of the developed reserve return where the divided (convenience) yield is:?? = ð ??”ð ??”(?? ? ??) ??**(7)**

Ito's lemma for ??(??, ??) is???? = ?? ?? ???? + 1 2 ?? ???? (????) 2 + ?? ?? ????**(8)**

Where the subscripts denotes partial derivatives, so that from (6)(????) 2 = ?? 2 ?? 2 ????**(9)**

so that???? = ?? ?? ???? + 1 2 ?? 2 ?? 2 ?? ???? ???? + ?? ?? ????**(10)**

The risk free portfolio values?? = ?? ? ???? = ?? ? ?? ?? ??

The quantity of stocks n to build a risk-free portfolio is the derivative of the option (named delta in financial market), because it makes the random term(????), of the return equation equal to zero. The portfolio returns per barrel = ???? ? ?? ?? (ð ??”ð ??”?????? + ???? ? ð ??”ð ??”??????) Equating with eqn (11) and substituting ???? in the equation (10)??(?? ? ?? ?? ??) = ?? ?? ???? + 1 2 ?? 2 ?? ???? ???? + ?? ?? ???? ? ?? ?? ð ??”ð ??”?????? ? ?? ?? ???? + ?? ?? ð ??”ð ??”??????

Simplifying this we get **??**6) putting (**??**6) into (15) we have,??? ?? = 1 2 ?? 2 ?? 2 ?? ???? + (?? ? ??)???? ?? ? ????**(**???????(??)? + 1 2 ?? 2 ???? = ????(??) ??(??)**(17)**????(??(??)) = ??? ? 1 2 ?? 2 ? ???? + ??????(??)**(18)**

Integrating from ?? 0 to ??, we have?????(??) ? ??(?? 0)? = ??? ? 1 2 ?? 2 ? ??? + ????(0,1)????

Since it follows from a normal distribution, therefore??(??) = ?? 0 ?????? ???? ? ? 1 2 ?? 2 ? ??? + ????(0,1)????? (19)

Hence, we have the equation for real simulation of the developed reserve . But for the risk-neutral simulation which we shall use, we have:??(??) = ?? 0 ?????? ???? ? ?? ? 1 2 ?? 2 ? ??? + ????(0,1)????? (**20**)

Where ?? ? = ?? ? ?? is the risk-neutral drift.

IV.

4 THE BLACK-SCHOLES' FORMULAE

Theorem: Let ?? ?? (?? 0 , ??, ??) be the fair price of a European call with strike price K, expiration T and initial asset price ?? 0 . Similarly, write ?? ?? (?? 0 , ??, ??) for the fair price of a European put with the same strike price K, expiration T and initial asset price ?? 0 . Then?? ?? (?? 0 , ??, ??) = S 0 N(d 1) ? ke ?rT ??(?? 2) ?? ?? (?? 0 , ??, ??) = ke ?rT N(?d 2) ? S 0 (?d 1)

Where??(??) = 1 ?2?? ? ?? ??? 2 2 ???? ?? ? ?? 1 = 1 ??? 2 ?? ???? ? ?? 0 ?? ???+ 1 2 ?? 2 ??? ?? ? ?? 2 = 1 ??? 2 ?? ???? ? ?? 0 ?? ???? 1 2 ?? 2 ??? ?? ? ?? 1 = 1 ??? 2 ?? ???? ? ?? 0 ?? ???+ 1 2 ?? 2 ??? ?? ?

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6 May

Growth Option Model For Oil Field Valuation

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Note that ?? 2 = ?? 1 ? ??? 2 ??. Since the fair price f of a contingent claim, with this underlying asset (the developed reserve value V) satisfies the B-S equation, that is, equation (12) Then, the corresponding B-S formulae for the European call and European put are given by ?? ?? (?? 0 , ??, ??, ??) = V 0 e ??T N(d 1)? ke ?rT ??(?? 2) ?? ?? (?? 0 , ??, ??, ??) = ke ?rT N(?d 2) ? V 0 e ??T (?d 1)

The Black-Scholes' expression for the fair price F (option value) of a contingent claim depends on; the asset value ?? 0 at time t = 0, the volatility ??, the time to maturity T, the interest rate r, and the strike price K. The sensitivities of the fair price F with respect to the first four parameters called Greeks are used for hedging.?? ?? (?? 0 , ??, ??, ??, ??, ??) = V 0 e ??T N ? 1 ?? 2 T In ? V 0 e ?r??+ 1 2 ? 2 ?T K ?? ? ke ?rT ?? ? 1 ??? 2 ?? ???? ? ?? 0 ?? (?????? 1 2 ?? 2)?? ?? ?? ?â??" ???? ?? ???? 0 = ?? ????? ? 1 ??? 2 ?? ???? ? ?? 0 ?? ??????+ 1 2 ?? 2 ??? ?? ?? + ?? 0 ?? ????? ?? ? ?? 1 2 2 ?? 0 ?2???? 2 ?? ? ???? ????? ?? ? ?? 2 2 2 ?? 0 ?2???? 2 ?? ?? â??" ?? 2 ?? ?? ???? 0 2 = ?? ????? ?? ? ?? 1 2 2 ?? 0 ???2?????? 0 + ???? ????? ?? ? ?? 2 2 2 ?? 0 2 ???2???? ?? â??" ???? ?? ???? = ?? 0 ?? ????? ?? ? ?? 1 2 2 ?2???? 2 ?? 1 2 ?? ? 3 2 ???? ?? 0 + 1 2 ??? ? ?? ? ?? 2 2 ? ?? ? 1 2 + 1 2 ?? ? 3 2 ???? ??? ???? 0 ?? ????? ??(**1**?(?? 2 ??) ????? ?? 0 + ??? ? ?? + ?? 2 2 ? ?? ? ???? ???) +?????? ????? ??(**1**?(?? 2 ??) ????? ?? 0 + ??? ? ?? + ?? 2 2 ? ?? ? ???? ???) ? ???? ????? ?? ? ?? 2 2 2 ?2???? 2 ?? 1 2 ?? ? 3 2 ???? ?? 0 + 1 2 ??? ? ?? ? ?? 2 2 ? ?? ? 1 2 + 1 2 ?? ? 3 2 ???? ??? = 1 2 ?? 0 ?? ????? ?? ? ?? 1 2 2 ???2???? ???? ? ?? ? ?? 2 2 ? + ???? ?? ?? 0 ?? ? ? ???? 0 ?? ????? ?? ? 1 ?(?? 2 ??) ????? ?? 0 ?? + ??? ? ?? + ?? 2 2 ? ???? +?????? ????? ??(**1**?(?? 2 ??) ????? ?? 0 ?? + ??? ? ?? ? ?? 2 2 ? ???) ? ???? ????? ?? ? ?? 2 2 2 2???2???? ? ???? ?? 0 ?? ?? + ??? ? ?? ? ?? 2 2 ??

Therefore, our ?? ?? , ?? ???? and ?? ?? for the European call is obtained as:?â??" ?? ?? = ?? ???(?????) ?? ? 1 ??? 2 (?? ? ??) ???? ? ?? 0 ?? ?????? + 1 2 ?? 2 ?(?????) ?? ?? + ?? 0 ?? ???(?????) ?? ? ?? 1 2 2 ?? 0 ?2???? 2 (?? ? ??) ? ???? ???(?????) ?? ? ?? 2 2 2 ?? 0 ?2???? 2 (?? ? ??) ?? â??" ?? ???? = ?? ???(?????) ?? ? ?? 1 2 2 ?? 0 ???2??(?? ? ??)?? 0 + ???? ???(?????) ?? ? ?? 2 2 2 ?? 0 2 ???2??(?? ? ??) ?? â??" ?? ?? = 1 2 ?? 0 ?? ???(?????) ?? ? ?? 1 2 2 ???2??(?? ? ??) ???? ? ?? ? ?? 2 2 ? + ???? ?? ?? 0 (?? ? ??) ? ? ???? 0 ?? ???(?????) ??(**1**?(?? 2 (?? ? ??)) ????? ?? 0 ?? + ??? ? ?? + ?? 2 2 ? (?? ? ??)?) + ?????? ???(?????) ??(**1**?(?? 2 (?? ? ??)) ????? ?? 0 ?? + ??? ? ?? ? ?? 2 2 ? (?? ? ??)?) ? ???? ???(?????) ?? ? ?? 2 2 2 2???2??(?? ? ??) ? ???? ?? 0 ?? (?? ? ??) + ??? ? ?? ? ?? 2 2 ?? V.

7 NUMERICAL EXAMPLE

In this section, we provide a numerical example of an oil company considering an investment in an oil company, the initial value, ?? 0 of the oil field is set at 1billion naira. An investment of 60million naira which could be thought of as the option premium on the option is required immediately for permitting and other preparations. This first stage will take one year. If this stage investment is made, then the firm may any time over the next five years choose to make a second stage investment of 800million naira to develop the reserve. The offshore lease is for 5 years. Set r = 0.03, ?? = 0.04 and ?? 2 = 0.0676. With these settings, we get the value of ?? (1) = 898, 783, 498.8??????????, ?? 1 = 0.9498, ?? 2 = 0.6898, ?? ?? = ?0.2571, ?? ???? = 2.3799 × 10 ?9 , ?? ?? = 57596310.04.

With this, the value of F for four years before expiration is ?? = 4, 162932, 089 and for one year before expiration is 1, 247, 975, 971 ?????????? which shows that in any case the option is profitable.

8 VI.

9 CONCLUSION

In this research work, we considered an investment opportunity of a firm using real options approach. We employed Geometric Brownian Motion to capture the value of the developed reserve and the classic model equation (12) to capture or obtain the value of the undeveloped reserve that is, the option value F. This option value F is also known as the fair price or theoretical value of the option. The value is to guide investors and managers in making rightful decisions rather than running into unnecessary risk. Real options approach is a very useful mathematical instrument. The investment is critically analyzed and we see that the investment is a lucrative one even with the imposition of some tight assumptions made. [1] [2] [3] [4] [5]

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