Global Journals LaTeX JournalKaleidoscope™

Artificial Intelligence formulated this projection for compatibility purposes from the original article published at Global Journals. However, this technology is currently in beta. *Therefore, kindly ignore odd layouts, missed formulae, text, tables, or figures.*

Studies on Image Segmentation Method Based On a New Symmetric Mixture Model with -K Means

Dr. Seshashayee M.[1] and Srinivasa Rao P.[2]

[1] GITAM UNiversity

Received: 8 August 2011 Accepted: 7 September 2011 Published: 22 September 2011

Abstract

In this paper we propose an image segmentation algorithm based on new-symmetric mixture model. Here the pixel intensities of the whole image are characterized through a newsymmetric mixture distribution, such that the statistical characteristics of the image coincide with that of the new symmetric distribution. Using the K-Means algorithm the number of image regions and initial estimates of the model parameters for the EM algorithm are obtained. The segmentation algorithm is proposed by component maximum likelihood under Bayesian frame work. The efficiency of the proposed method is studied with the five images taken from the Berkeley image dataset and computing the values image segmentation measures like global consistency error, probabilistic rand index and variation of information. A comparative study of the proposed model with Gaussian mixture model reveals that the proposed method performs better. The efficiency of the proposed method with respect to the image retrieval is also studied.

Index terms— Image Segmentation, New Symmetric Mix- ture Model, Image Quality Metrics, Kmeans algorithm, EM algorithm.

1 INTRODUCTION

mage segmentation is a preprocessing step in image analysis and understanding. Much work has been reported in literature regarding image segmentation. Pal S.K. and Pal N.R (1993), Jahne (1995), Cheng et al (2001), Mantas Paulinas and Audrius Usinskas ??2007) and Shital Raut et al (2009) have discussed various image segmentation methods.

The image segmentation methods are usually classified into three categories namely (i) segmentation methods based on histogram, threshold and edge based techniques, (ii) model based image segmentation methods and (iii) image segmentation based on other methods like graph, saddle point, neural networks, fuzzy logic etc., (Caillol H. et al (1993) In Gaussian mixture model the whole image is characterized by the collection of several image regions, where each region is characterized by a Gaussian distribution. That is the pixel intensities in each image region follow a Gaussian distribution. This Gaussian assumption serves well only when the pixel intensities in each image region are meso-kurtic and symmetric. But in some images like natural scenes the pixel intensities of the image region may not be meso-kurtic even though they are symmetric. Hence to have an accurate analysis of the images, it is needed to develop image segmentation methods based on Non-Gaussian mixture models.

In Non-Gaussian symmetric mixture models the kurtosis plays a dominant role. Based on the kurtosis the Non-Gaussian models can be classified into two categories platy-kurtic and lepto-kurtic. In general many of the natural scenes will have image regions having platy-kurtic nature. That is the kurtosis of the pixel intensities in the image regions is less than three. One such model available in literature is new-symmetric distribution given by Srinivasa Rao K. et al ??1997). The new-symmetric distribution is having kurtosis 2.52 and symmetric. So it is a platy-kurtic distribution. Hence to have an efficient image segmentation algorithm for images having platy-kurtic distributed pixel intensities in the image regions, we develop and analyze an image segmentation algorithm based on new-symmetric mixture model.

For developing the image segmentation algorithm we require the number of components in the image. This is obtained from K-means algorithm. The initial estimates of the model parameters are obtained from the moment estimates. The updated equations for estimating the model parameters through the EM algorithm are derived. The segmentation algorithm is also presented by taking component maximum likelihood. The efficiency of the proposed algorithm is studied through experimentation. intensity z = f(x , y) is a random variable, because of the fact that the brightness measured at a point in the image is influenced by various random factors like vision, lighting, moisture, environmental conditions etc,. To model the pixel intensities of the image region it is assumed that the pixel intensities of the region follow a new symmetric distribution given by Srinivasa Rao K. et al., **??**1997). The probability density function of the pixel intensity is (1) The probability curve of new symmetric distribution is shown in Figure **??**.

2 Its central moments are and

(2)

The kurtosis of the distribution is

where, K is number of regions , 0 ? i ? ? 1 are weights such that ? i ? = 1 and is as given in equation (1). i ? is the weight associated with ith region in the whole image.

In general the pixel intensities in the image regions are statistically correlated and these correlations can be reduced by spatial sampling (Lie.T and Sewehand. W(1992)) or spatial averaging **??** Kelly P.A. et al.(1998)) . After reduction of correlation the pixels are considered to be uncorrelated and independent. The mean pixel intensity of the whole image is . III. II.

3 ESTIMATION OF THE MODEL PARAMETERS BY EM ALGORITHM

4 FINITE MIXTURE OF NEW SYMMETRIC DISTRIBUTION

In low level image analysis the entire image is considered as a union of several image regions. In each image region the image data is quantized by pixel intensities. For a given point (pixel) (x , y), the pixel2 2 2 1 2 2 (, ,) , , , 0 3 2 z z e f Z Z µ µ ? ? µ ? µ ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? + ? ? ? ? ? ? ? ? ? ? ? = ?? < < ? ?? < < ? > Figure 1 : Probability curve of new symmetric distribution 2 3 1 ( ) ( ) 2 2 2 2 (3 / 2) n n n n n µ ? ? ? ? + Î?" + ? ? = ? ? ? ? ? ? 2 1 0 n µ + = 2.52 2 ? = 2 ( ) (/) , 1 K p z f z i i i i i ? µ ? = ? = 2 (, ,) i f z µ ? ( ) 1 K E Z i i i ? µ = ? = . N ( ) 1 ( ) (,) l s L p z s ? ? = = ? . N 1 ( ) (,)**1**K L f z s i i i s ? ? ? = ? ? = ? ? ? = ? ? ? log ( ) log (,)**1**

1N K L f z s i i i s i ? ? ? ? ? = ? ? ? ? = = ? ? , 2

(, , ; 1, 2,...,)i i i i K ? µ ? ? = = 2 1 2 2 2 log ( ) log 3 2 1 1 s z i z s i i e i i N K L s i i µ ? µ ? + = ? ? = = ( ) ( ) ; l Q ? ? [ ] ( ) log ( ) / l E L z ? ? ( ) ( ) ()( ) ( )**1 1**

;

(,) log (,) logK N l l i s i s i i s Q t z f z ? ? ? ? ? = = = + ??**(6)**

The updated equation of ? for (l +1) th th estimate is ( )1 ( ) 1 1 (,) N l l i i s s t z N ? ? + = = ?

5 October

The entire image is a collection of regions which are characterized by new symmetric distribution. Here, it is assumed that the pixel intensities of the whole image follow a K -component mixture of new symmetric distribution and its probability density function is of the form.

6 The first step of the EM algorithm requires the estimation of the likelihood function of the sample observations. function of the sample is

The expectation of the log likelihood (7) The updated equation of ? at (l +1) th iteration is where, =

The updated equation of2 i ? at (l +1) th iteration is**(9)**

where IV.

7 INITIALIZATION OF THE PARAMETERS BY K -MEANS

The efficiency of the EM algorithm in estimating the parameters is heavily dependent on the number of regions in the image. The number of mixture components initially taken for K -Means algorithm is by plotting the histogram of the pixel intensities of the whole image. The number of peaks in the histogram can be taken as the initial value of the number of regions K.

The mixing parameters **??**2000). This method performs well if the sample size is large and its computational time is heavily increased. When the sample size is small, some small regions may not be sampled. To overcome this problem we use the K -Means algorithm to divide the whole image into various homogeneous regions. In K -Means algorithm the of the clusters are recomputed as soon as the pixel joins a cluster.

After determining the final values of K (number of regions) , we obtain the initial estimates of 2 , i i µ ? and i ? for the i th region using the segmented region pixel intensities with the method given by Srinivasa Rao et al., **??**1997) for new symmetric distribution .The initial estimate i ? is taken as1 K i ? =

, where i = 1,2,...,K. V.

8 The parameters

9 SEGMENTATION ALGORITHM

In this section, we present the image segmentation algorithm. After refining the parameters the prime step in image segmentation is allocating the pixels to the segments of the image. This operation is performed by Segmentation Algorithm. The image segmentation algorithm consists of four steps.

Step 1) Plot the histogram of the whole image.

Step 2) Obtain the initial estimates of the model parameters using K-Means algorithm and moment estimators as discussed in section 4

Step 3) Obtain the refined estimates of the model parameters 2 , i i µ ? and i ? for i=1,2,...,K by using the EM algorithm with the updated equations

Step 4) Assign each pixel into the corresponding j th region (segment) according to the maximum likelihood of the j th component L j.

That is , VI.

10 EXPERIMENTAL RESULTS

To demonstrate the utility of the image segmentation algorithm developed in this chapter, an experiment is conducted with five images taken from Berkeley images dataset (http://www.eecs.berkeley .= ( ) ( ) ( ) ( ) 1 1 (,) 1 (,) l l N i i s K l l s i i s i f z N f z ? ? ? ? = = ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ( ) ( ) 2( ) ( ) ( ) ( ) 2 2( ) ( ) 1 1 (1) ( ) 1 2 (,)**(,) 2 (** ,

) The initial estimates of the number of the regions K in each image are obtained and given in Table 1. From Table 1, we observe that the image HORSE has two segments, images TOWER and BIRD have three segments each and images **??**AN l l N N i s i l l s i s i s l l s s i s i l i N l i s s z z t z t z z t z ? µ ? ? ? µ µ ? = = + = ? ? ? ? ? ? ? ? + ? ? ? = ? ? ? ( ) (,) l i s t z ? ( ) ( ) ( ) (1) ( )**2**( ) (1) ( )**2 1 (, ,) (,)**l l l i i s i i K l l l i i s i i i f z f z ? µ ? ? µ ? + + = ?**(8) ( ) ( ) ( ) ( ) ( )**( ) 2 (1) 2 ( ) 2 2 2( )**(1) 1 (1) 2** () 1 1 2 ( ) (,) 2 2 (,) i l N i l l s i i s l l s i s l i N l i s s z t z z t z ? µ ? ? µ ? ? + + = + = ? ? ? ? ? ? ? ? ? ? + ? ? ? ? ? = ? ? ( ) ( ) ( ) (1) (1) 2 ( ) ( ) (1)**(1) 2 1 (, ,) (,) (, ,)**l l l i i s i i l i s K l l l i i s i i i f z t z f z ? µ ? ? ? µ ? + + + + = = ? 2 1 j 2 1 2 L (3 2) 2 max s j j s j j j k j z z e µ ? µ ? = + ? , , s z ?? < < ? , 0 j j µ ? ?? < < ? > edu/? + + ? ? ? ? ? ? = ? ? ? ? ? ? ? ? ? ? ? ? + ( )**2**? + ? ? ? ? ? ? ? ? ? = ? ? ? ? + ? ? ? ? ? ? ? ? ?**2**? + ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? + ? ? ? ? ? ? ? ? ? 32.7780)(3) 2? ( )**2**? + ? ? ? ? ? ? ? ? ? = ? ? ? ? + ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? + ? ? ? ? ? ? ? ?

The estimated probability density function of the pixel intensities of the image BOAT is ( )**2 ( ) 2 2**? + ? ? ? ? ? ? ? ? ? = ? ? ? ? + ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? + ? ? ? ? ? ? ? ? ? ? ? ? ? ? + ? ? ? ? ? ? ? ? ? 2 73 (29.6973)(3) 2? ? ? ? ? ? ? ? ?

The estimated probability density function of the pixel intensities of the image TOWER is ( )**2 ( ) 2 2**? + ? ? ? ? ? ? ? ? ? = ? ? ? ? + ? ? ? ? ? ? ? ? ?

11 PERFORMANCE EVALUTION

After conducting the experiment with the image segmentation algorithm developed in this chapter, its performance is studied. The performance evaluation of the segmentation technique is carried by obtaining the three performance measures namely, (i) Probabilistic Rand Index (PRI), (ii) Variation Of Information (VOI) and (iii) Global Consistence Error (GCE). The performance of developed algorithm using finite new symmetric distribution mixture model (NSMM-K) is studied by computing the segmentation performance measures namely PRI, GCE, and VOI for the five images under study. The computed values of the performance measures for the developed algorithm and the earlier existing finite Gaussian mixture model(GMM) with K-Means algorithm are presented in Table 3 for a comparative study. From table 3 it is observed that the PRI values of the proposed algorithm for the five images considered for experimentation are less than that of the values from the segmentation algorithm based on finite Gaussian mixture model with K-means. Similarly GCE and VOI values of the proposed algorithm are less than that of Finite Gaussian Mixture Model. This reveals that the proposed algorithm outperforms the existing algorithm based on the finite Gaussian mixture model. When the kurtosis parameter of each component of the model is zero, the model reduces to finite Gaussian mixture model and even in this case the algorithm performs well.

After developing the image segmentation method it is needed to verify the utility of segmentation in model building of the image for image retrieval.The performance evaluation of the retrieved image can be done by subjective image quality testing or by objective image quality testing. The objective image quality testing methods are often used since the numerical results of an objective measure allows a consistent comparison of different algorithms. There are several image quality measures available for performance evaluation of the image segmentation method. An extensive survey of quality measures is given by Eskicioglu A.M. and Fisher P.S. (1995). For the performance evaluation of the developed segmentation algorithm, we consider the image quality measures like average difference, maximum distance, image fidelity, mean square error, signal to noise ratio and image quality index.

Using the estimated probability density functions of the images under consideration the retrieved images are obtained and are shown in Figure 4. From the Table 4, it is observed that all the image quality measures for the five images are meeting the standard criteria. This implies that using the proposed algorithm the images are retrieved accurately. A comparative study of proposed algorithm with that of algorithm based on Finite Gaussian Mixture Model reveals that the MSE of the proposed model is less than that of the finite Gaussian mixture model. Based on all other quality metrics also it is observed that the performance of the proposed model in retrieving the images is better than the finite Gaussian mixture model.

12 VIII.

13 CONCLUSION

An image segmentation algorithm based on new symmetric mixture model with K-means is developed and evaluated. This algorithm is more suitable for the images having platy-kurtic image regions. The new symmetric mixture model is capable of characterizing several natural images with kurtosis close to 2.52. The updated equations of the model parameters are derived through EM algorithm under Bayesian framework. The estimated probability density function of the pixel intensities in the whole image is useful for the image retrieval. The experimental results revealed that the proposed method out performs the existing Gaussian mixture model in both image segmentation and image retrieval. [1] [2] [3] [4]

Figure 1:

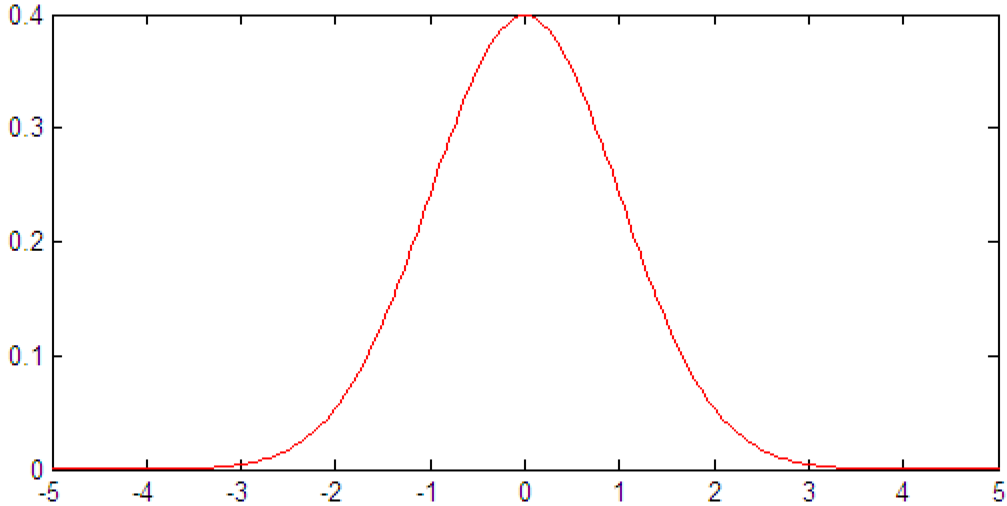


Figure 2:

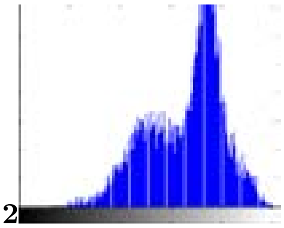


Figure 3: i ? and the model parameters µ i , 2 i?

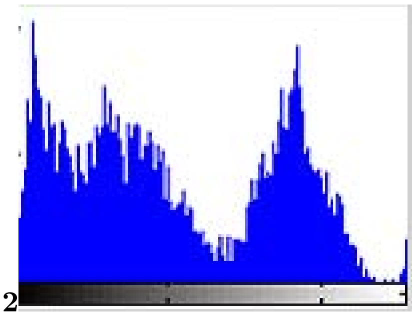


Figure 4: i µ and 2 i?

182

[1]© 2011 Global Journals Inc. (US)

[2]Studies on Image Segmentation Method Based On a New Symmetric Mixture Model with K -Means © 2011 Global Journals Inc. (US)

[3]OctoberStudies on Image Segmentation Method Based On a New Symmetric Mixture Model with K -Means

[4]© 2011 Global Journals Inc. (US) Studies on Image Segmentation Method Based On a New Symmetric Mixture Model with K -Means

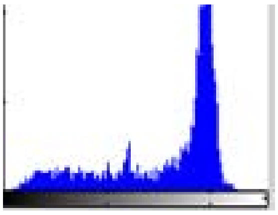


Figure 5:

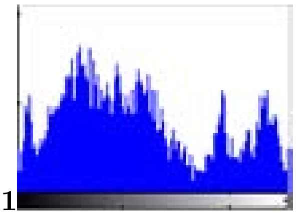
2011 

Figure 6: 2011 October.

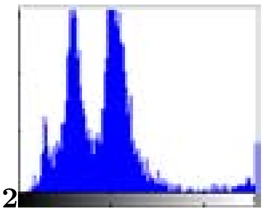
2 

Figure 7: Figure 2 .

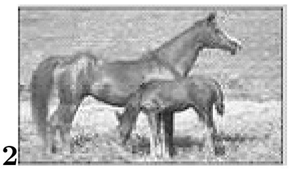
2 

Figure 8: Figure 2 :

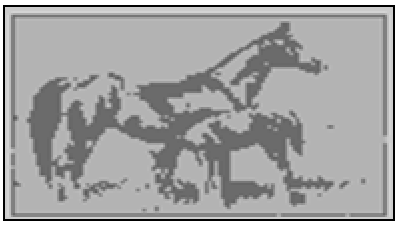


Figure 9:

Figure 10:

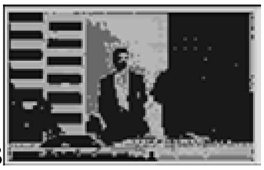
3 

Figure 11: Figure 3 :

4 

Figure 12: Figure 4 :

1

IMAGE	HORSE	MAN	BIRD	BOAT	TOWER
Estimate of K	2	4	3	4	3

Figure 13: Table 1 :

:

Table : 2.d

Estimated Values of the Parameters for BOAT Image

2.b Estimated Values of the Parameters for MAN Image Number of Image Regions (K =4) Parameters Esti

Estimated Values of the Parameters for BIRD Image

		Number of Image Regions (I			
		Estimation of Initial			Estimation of Final Param by EM Algor
Parameters		Parameters			
		Regions(i)			Regio
		1	2	3	1
?	i	1/3	1/3	1/3	0.13161 0.6678
µ	i	53.491	124.05	124.05	60.691

2 i The estimated probability density function of the pixel ? 535.4 513.93 513.93 857.07 86.799 1581.2

intensities of the image BIRD is

[Note: c Global Journal of Computer Science and Technology Volume XI Issue XVII Version I 54 2011 October different parameters. The number of segments in each of the five images considered for experimentation is determined by the histogram of pixel intensities. The histograms of the pixel intensities of the five images]

Figure 14: Table :

3

IMAGES	PERFORMACE METHOD PRI	MEASURES GCEVOI
HORSE	GMM NSMM-K 0.9283 0.1634 1. 8403 0.9142 0.1737 1.8643	
MAN	GMM NSMM-K 0.9342 0.1734 1.7875 0.9228 0.3107 1. 8389	
BIRD	GMM NSMM-K 0.9140 0.1352 1.7259 0.9106 0.1369 1. 7479	
BOAT	GMM NSMM-K 0.9174 0.6483 1.7542 0.9026 0.6485 1. 7882	
TOWER	GMM NSMM-K 0.9246 0.0981 1.7988 0.9102 0.1090 1. 8643	

Figure 15: Table 3 :

4

IMAGE	Quality Metrics	FGMM	FNSDMM with K-Means	Standard Limits
	Average Difference	0.5011	0.44135	Close to 1
	Maximum Distance	1.0000	1.0000	Close to 1
HORSE	Image Fidelity Mean Square Error	1.0000 0.5011	1.0000 0.4414	Close to 1 Close to 0
	Signal to Noise Ratio	5.6542	5.9301	As big as possible
	Image Quality Index	1.0000	1.0000	Close to 1
	Average Difference	0.4858	0.50021	Close to 1
	Maximum Distance	1.0000	1.0000	Close to 1
MAN	Image Fidelity Mean Square Error	1.0000 0.4995	1.0000 0.5079	Close to 1 Close to 0
	Signal to Noise Ratio	5.6828	5.6251	As big as possible
	Image Quality Index	1.0000	1.0000	Close to 1
	Average Difference	0.4939	0.6573	Close to 1
	Maximum Distance	1.0000	1.0000	Close to 1
BIRD	Image Fidelity	1.0000	1.0000	Close to 1
	Mean Square Error	0.8590	0.5050	Close to 0
	Signal to Noise Ratio	5.6861	4.4842	As big as possible
	Image Quality Index	1.000	1.0000	Close to 1
	Average Difference	0.5039	0.6217	Close to 1
	Maximum Distance	1.0000	1.0000	Close to 1
BOAT	Image Fidelity Mean Square Error	1.0000 0.7931	1.0000 0.5070	Close to 1 Close to 0
	Signal to Noise Ratio	5.6318	4.6573	As big as possible
	Image Quality Index	1	1.0000	Close to 1
	Average Difference	0.4936	0.6640	Close to 1
	Maximum Distance	1.0000	1.0000	Close to 1
	Image Fidelity	0.9999	0.9999	Close to 1
TOWER	Mean Square Error	0.8788	0.5076	Close to 0
	Signal to Noise Ratio	5.6870	4.4347	As big as possible
	Image Quality Index	1.0000	1.0000	Close to 1

Figure 16: Table 4 :

[October Transactions on Image Processing] , *October Transactions on Image Processing* 1 p. .

[Martin et al. ()] 'A database of human Segmented natural images and its application to evaluating segmentation algorithms and measuring ecological statistics'. D Martin , C Fowlkes , D Tal , J Malik . *proceedings of 8th International Conference on Computer vision*, (8th International Conference on Computer vision) 2001. 2 p. .

[Bilmes ()] *A Gentle Tutorial of the EM Algorithm and its application to Parameter Estimation for Gaussian Mixture and Hidden Markov Models*, Jeff A Bilmes . ICSI-TR-97-021. 1997. University of Berkeley (Technical Report)

[Jahne ()] *A Practical Hand Book on Image segmentation for Scientific Applications*, Jahne . 1995. CRC Press.

[Pal and Pal ()] 'A Review On Image Segmentation Techniques'. S K Pal , N Pal . *Pattern Recognition* 1993. 26 (9) p. .

[Xu ()] 'A segmentation algorithm for noisy images, design and evaluation'. Y Xu . *Pattern Recognition letters* 1998. 19 p. .

[Paulinas and Usinskas ()] 'A survey of genentic algorithms applications for image enhancement and segmentation'. Mantas Paulinas , Andrius Usinskas . *Information Technology and control* 2007. 36 (3) p. .

[Kelly ()] 'Adaptive segmentation of speckled images using a hierarchical random field model'. P A Kelly . *IEEE Transactions Acoust. Speech. Signal Processing* 1988. 36 (10) p. .

[Rose and Turi ()] *Cluster Based Image Segmentation*, H Rose , Turi . 2001. Australia. Monash University (phd Thesis)

[Cheng ()] *Color Image Segmentation: Advances and Prospects" Pattern Recognition*, Cheng . 2001. 34 p. .

[Meila ()] 'Comparing Clustering -An axiomatic view'. M Meila . *proceedings of the 22nd International Conference on Machine Learning*, (the 22nd International Conference on Machine Learning) 2005. p. .

[Zhang ()] 'EM Algorithms for Gaussian Mixtures with Split-and-merge Operation'. Z H Zhang . *Pattern Recognition* 2003. 36 (9) p. .

[Caillol ()] 'Fuzzy Random Fields and Unsupervised Image Segmentation'. H Caillol . *IEEE Transactions on Geo-Science and Remote Sensing*, 1993. 31 p. .

[Qian ()] *Gaussian Mixture Model For Relevance Feedback In Image Retrieval*, Fang Qian . 2002. IEEE.

[Global Journal of Computer Science and Technology Volume XI Issue XVII Version I 58] *Global Journal of Computer Science and Technology Volume XI Issue XVII Version I 58*,

[Eskicioglu and Fisher ()] 'Image Quality Measures and their Performance'. M Eskicioglu , P Fisher . *IEEE Transactions On Communications* 1995. 43 (12) .

[Tolias and Pamas ()] 'Image Segmentation by a Fuzzy clustering algorithm using adaptive spatially constrained membership functions'. Y A Tolias , S M Pamas . *IEEE Transactions on Image Processing*, 1998. p. .

[Raut ()] 'Image segmentation-A State-Of-Art survey for Prediction'. Shital Raut . *International conference on Adv. Computer control* 2009. p. .

[Yamazaki ()] 'Introduction of EM algorithm into color image segmentation'. T Yamazaki . *Proceedings of ICIRS'98*, (ICIRS'98) 1998. p. .

[Mclanchlan and Krishnan ()] G Mclanchlan , T Krishnan . *The EM Algorithm and Extensions*, (New York) 1997. 1997. John Wiley and Sons.

[Mclanchlan and Peel ()] G Mclanchlan , Peel . *The EM Algorithm For Parameter Estimations*, (New York) 2000. 2000. John Wiley and Sons.

[Rao et al. ()] 'On a New Symmetrical Distribution'. Srinivas Rao , K , C V S R Vijay Kumar , J Lakshmi Narayana . *Journal of Indian Society for Agricultural Statistics* 1997. 50 (1) p. .

[Lie ()] 'Performance evaluation of Finite normal mixture model based image segmentation'. T Lie . *IEEE Transactions on Image processing* 1993. 12 (10) p. .

[Lie and Sewehand ()] 'Statistical approach to X-ray CT imaging and its Applications in image analysis'. Lie , Sewehand . *IEEE Trans. Med. Imag* 1992. 11 (1) p. .

[Unnikrishnan ()] 'Toward objective evaluation of image segmentation algorithms'. R Unnikrishnan . *IEEE Trans.Pattern Analysis and Machine Intelligence* 2007. 29 (6) p. .

[Yiming ()] 'Unsupervised Color Image Segmentation Based On Gaussian Mixture Models'. W U Yiming . *Proceedings Of 2003 Joint Conference At The 4th International Conference On Information, Communication and Signal Processing*, (Of 2003 Joint Conference At The 4th International Conference On Information, Communication and Signal Processing) 2003. 1 p. .

[Srinivas and Rao ()] 'Unsupervised image segmentat using finite doubly truncated Gaussian mixture model and Hierarchial clustering'. Y Srinivas , Srinivas Rao , K . http://www.eecs.berkeley.edu/Research/Projects/CS/vision/bsds/BSDS300/html/dataset/images.html *The Berkeley segmentation dataset*, 2007. 93 p. .

[Nasios and Bors ()] 'Variational learning for Gaussian Mixtures'. N Nasios , A G Bors . *IEEE Transactions on Systems, Man , and Cybernatics -Part B : Cybernatics, Vol36*, 2006. p. .